# Towards a Less Conservative Theory of Machine Learning

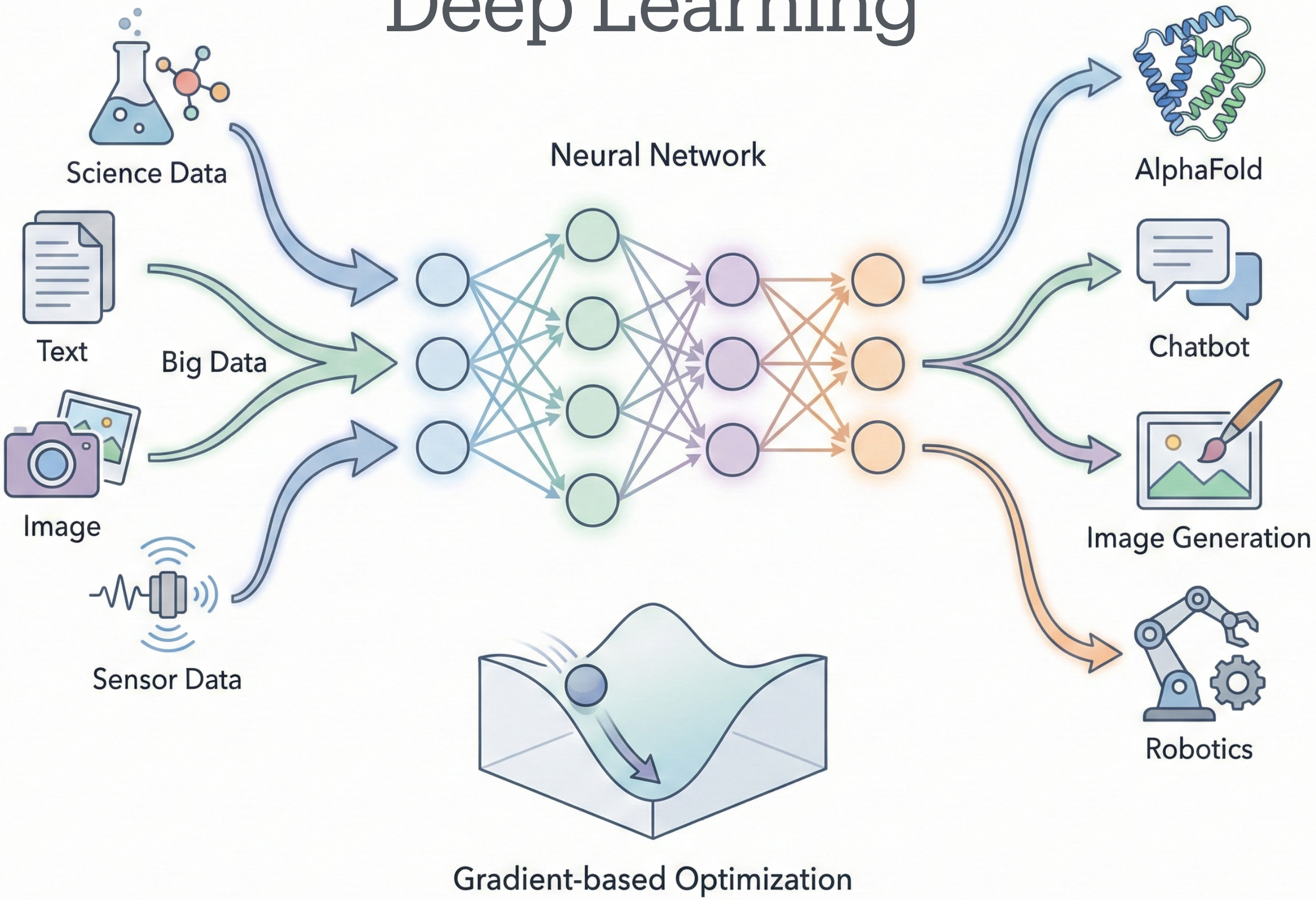
## Unstable Optimization & Implicit Regularization

Jingfeng Wu

Simons Institute for the Theory of Computing

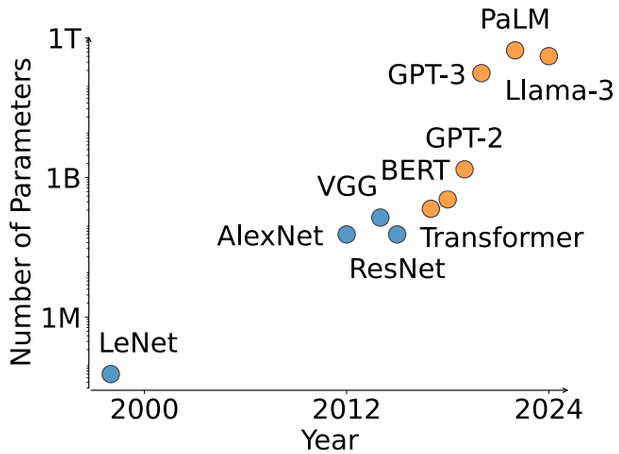
Berkeley UNIVERSITY OF CALIFORNIA

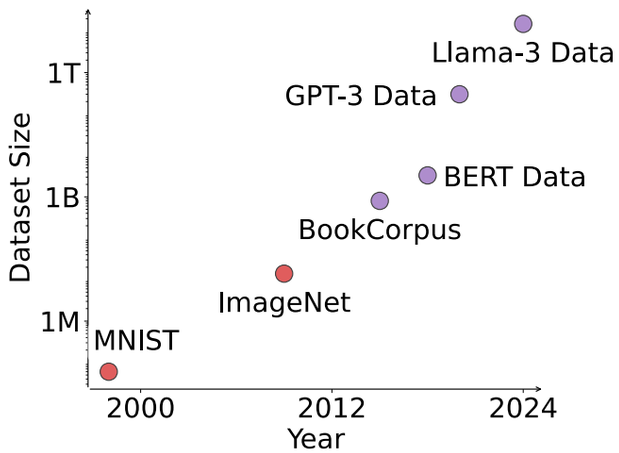
# Deep Learning



# What makes deep learning thrive?

## scaling



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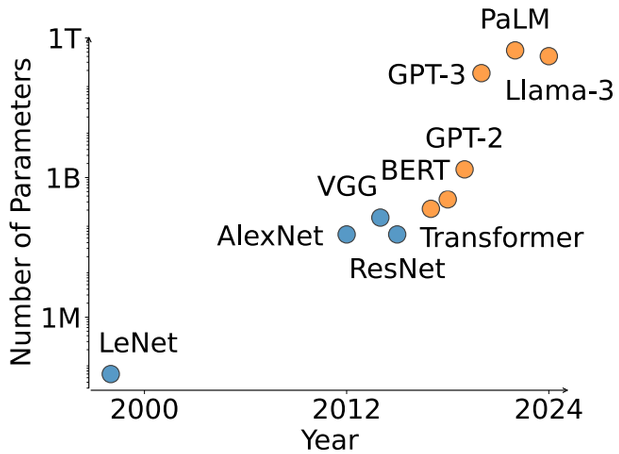


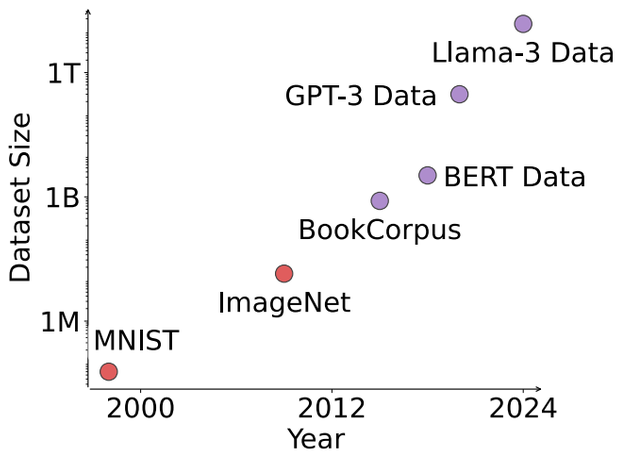


## new mechanisms

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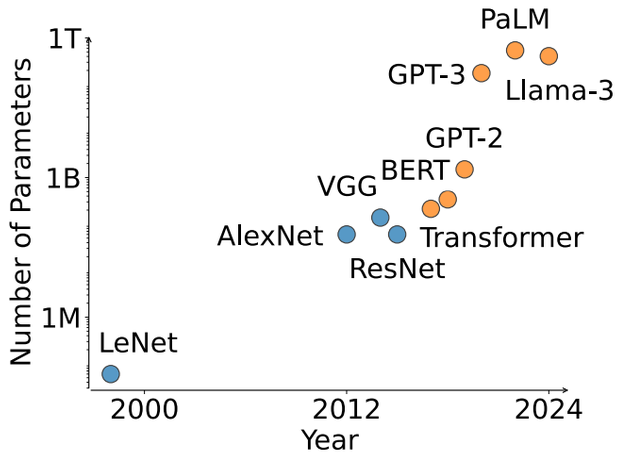


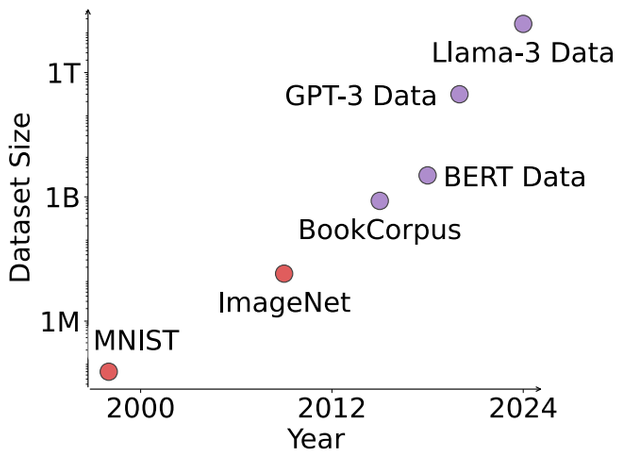


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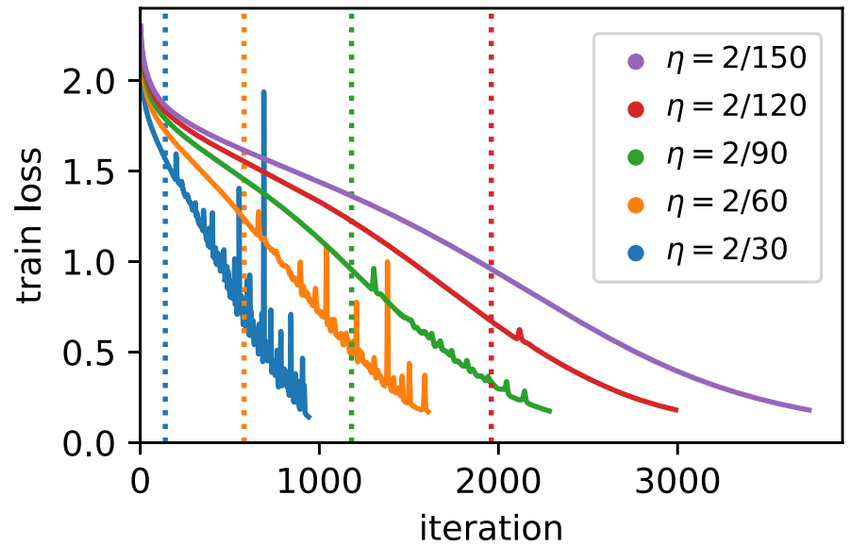

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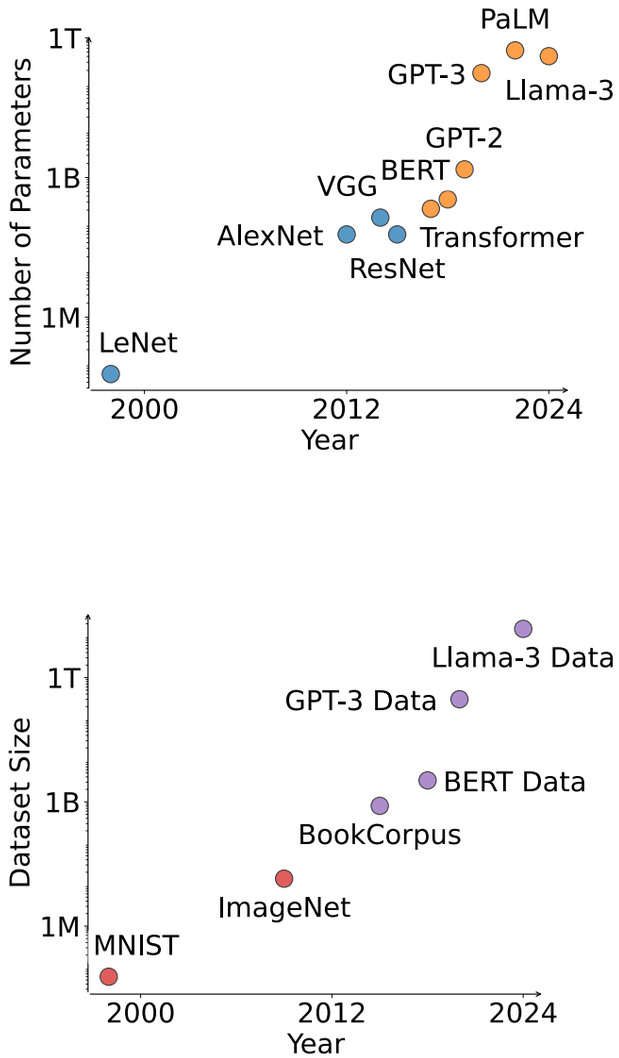


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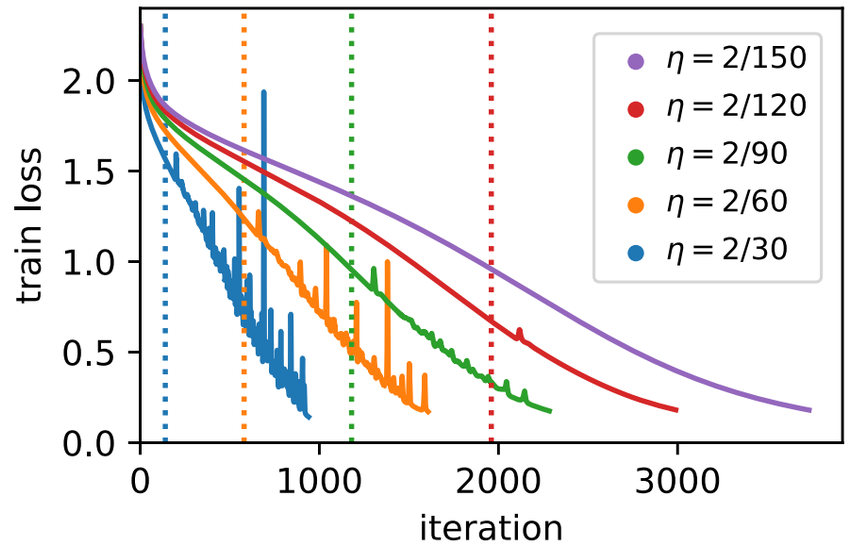

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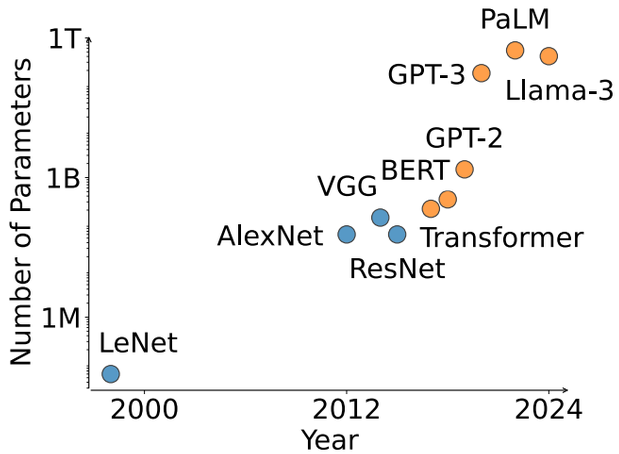
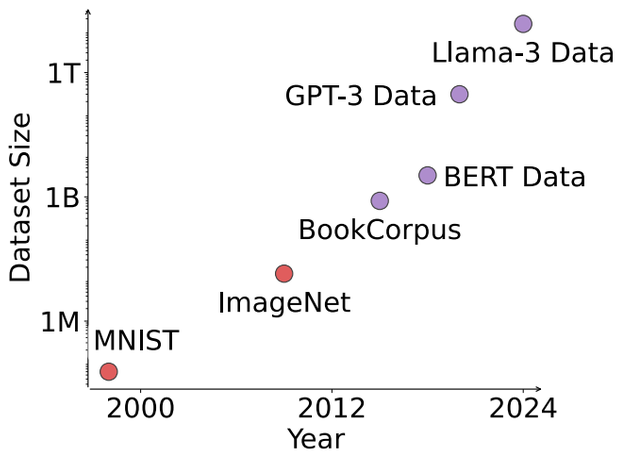
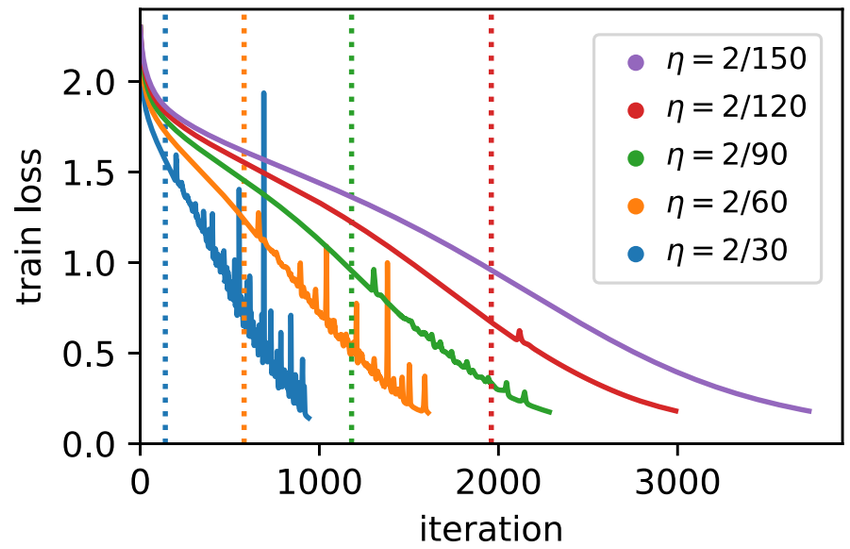



## new mechanisms

optimization

- training instability
- "edge of stability"
- ...

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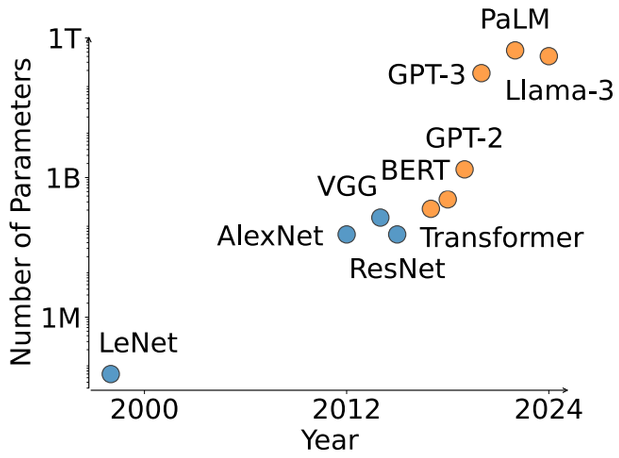


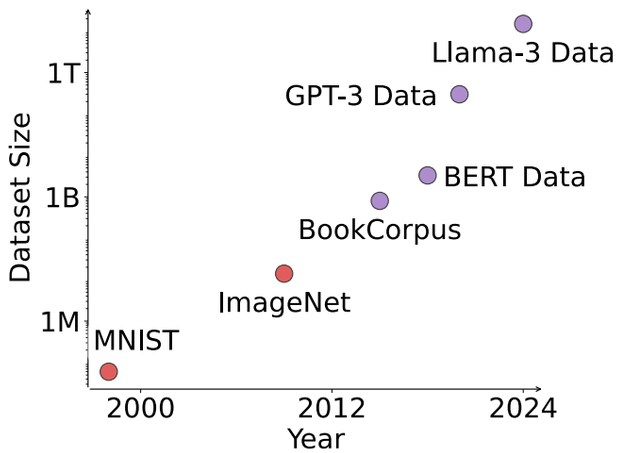


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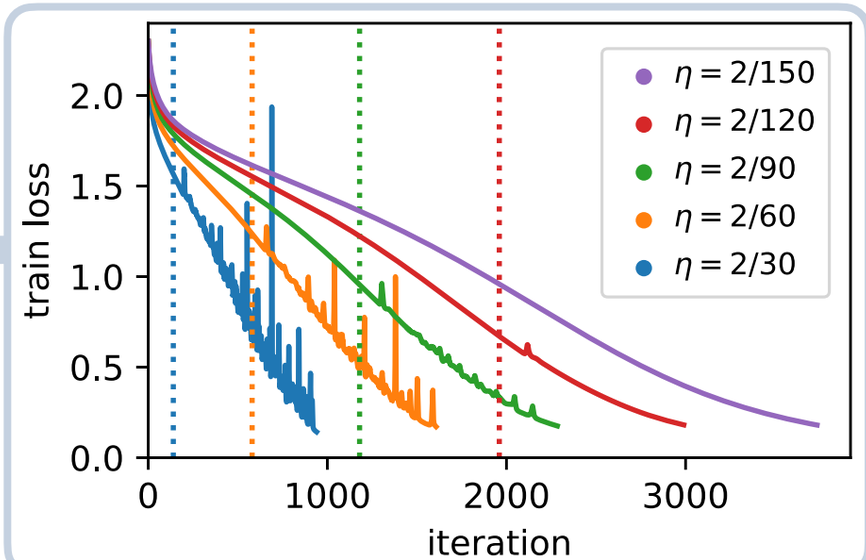


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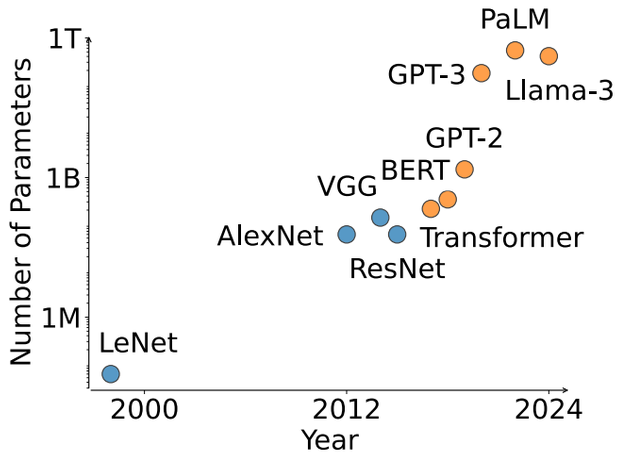


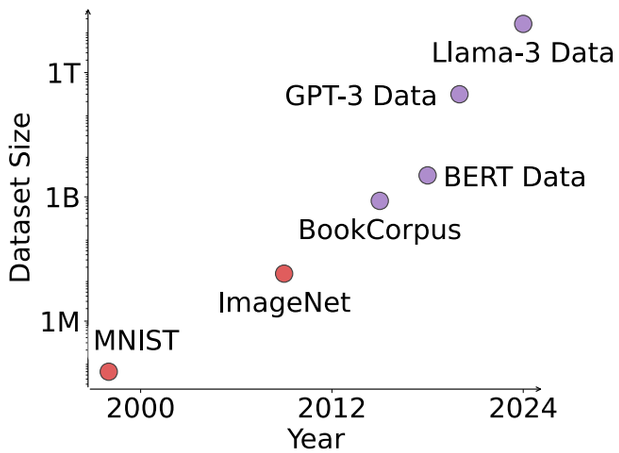


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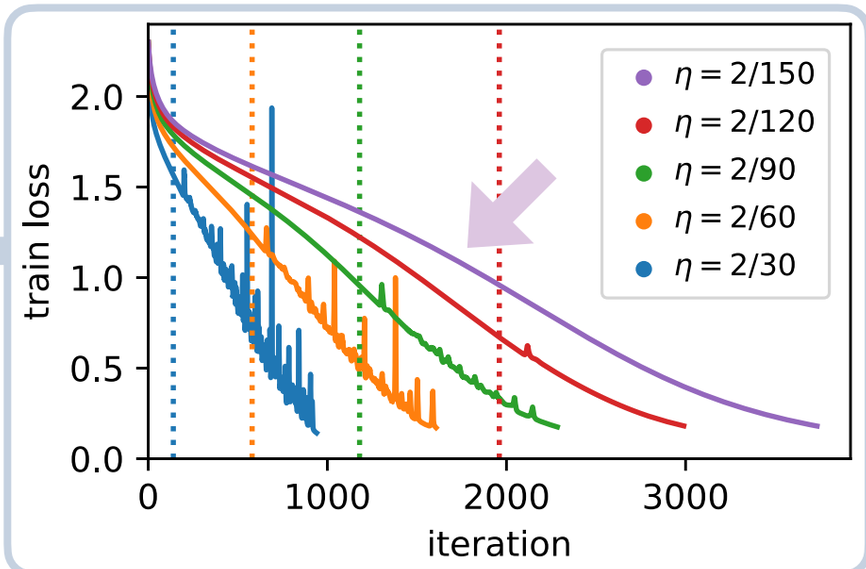


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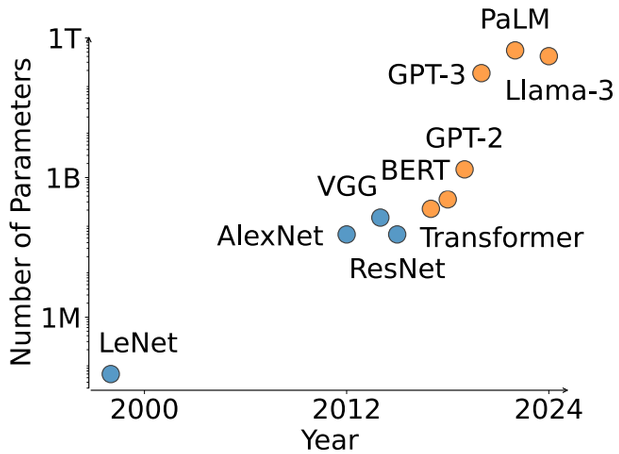


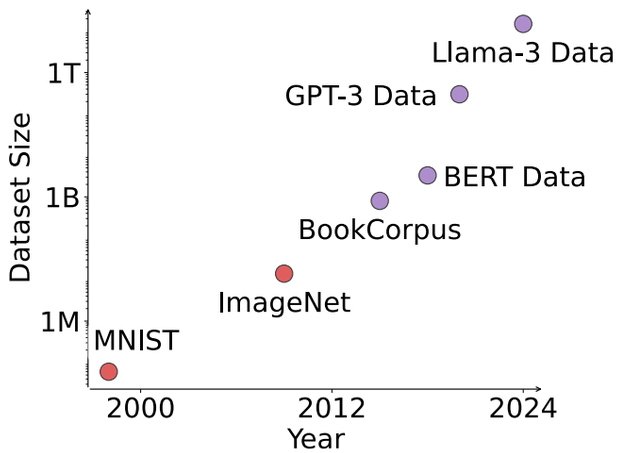


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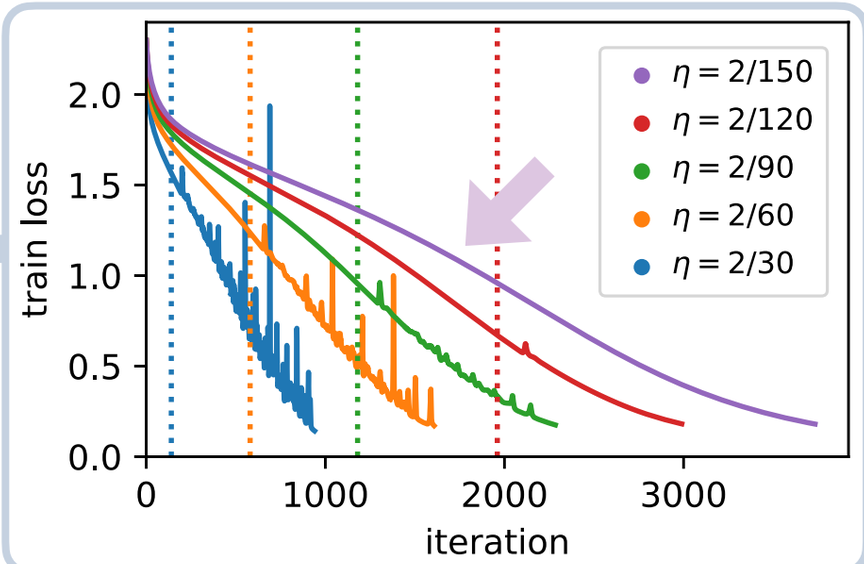
large stepsize
unstable optimization

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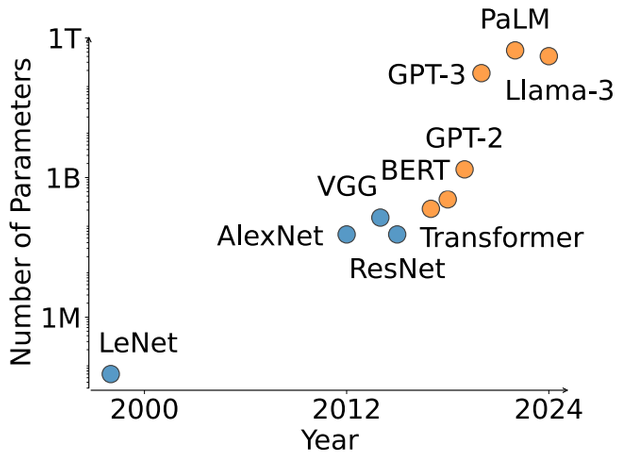


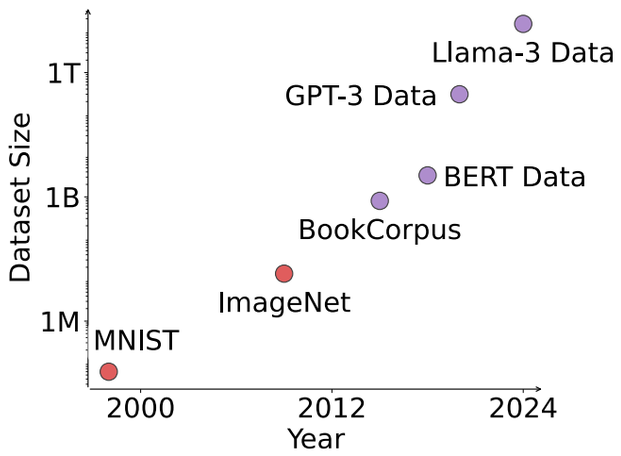


## new mechanisms

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"edge of stability"

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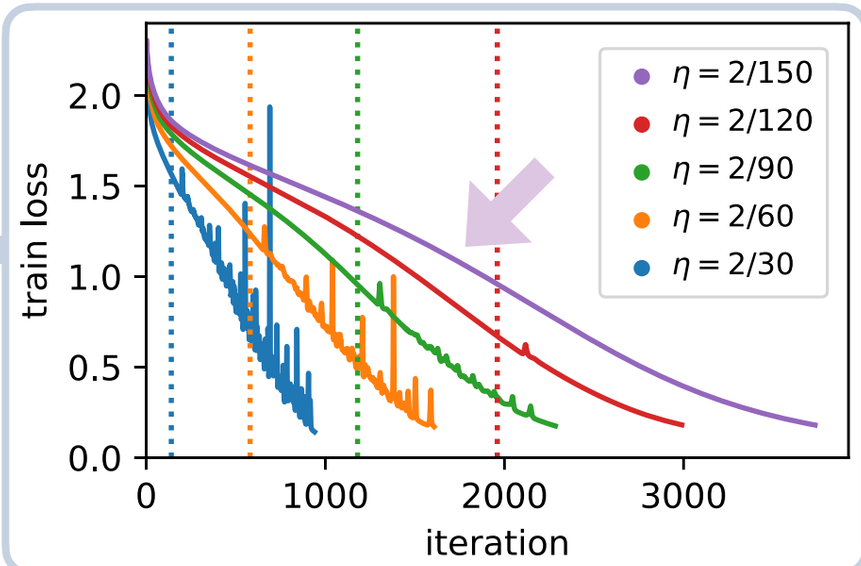
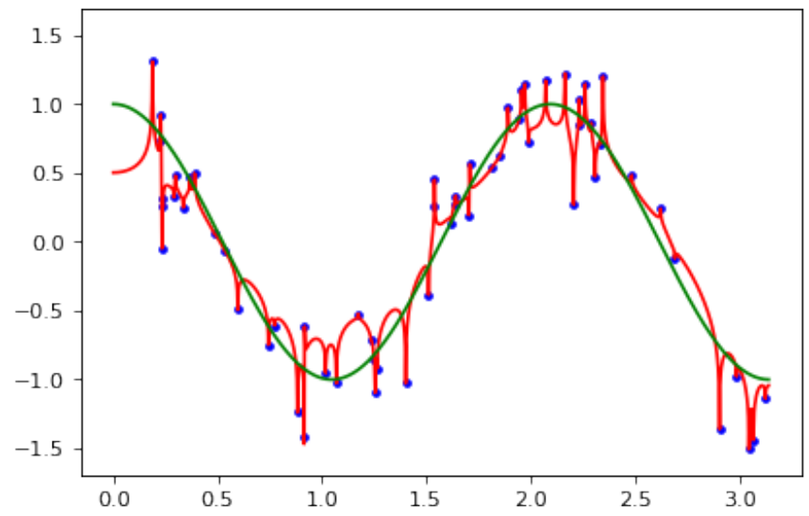
large stepsize unstable optimization




### generalization

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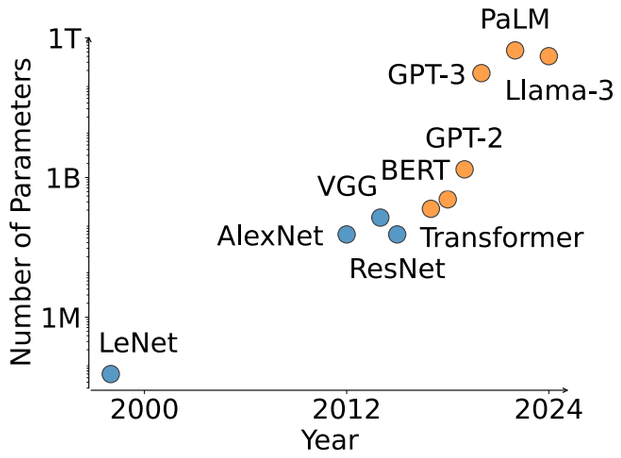


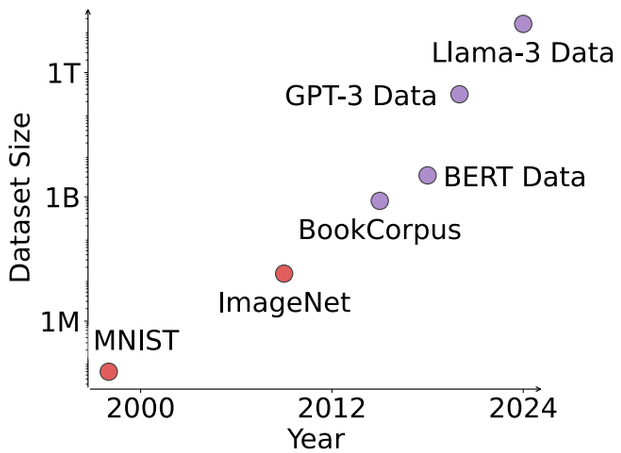


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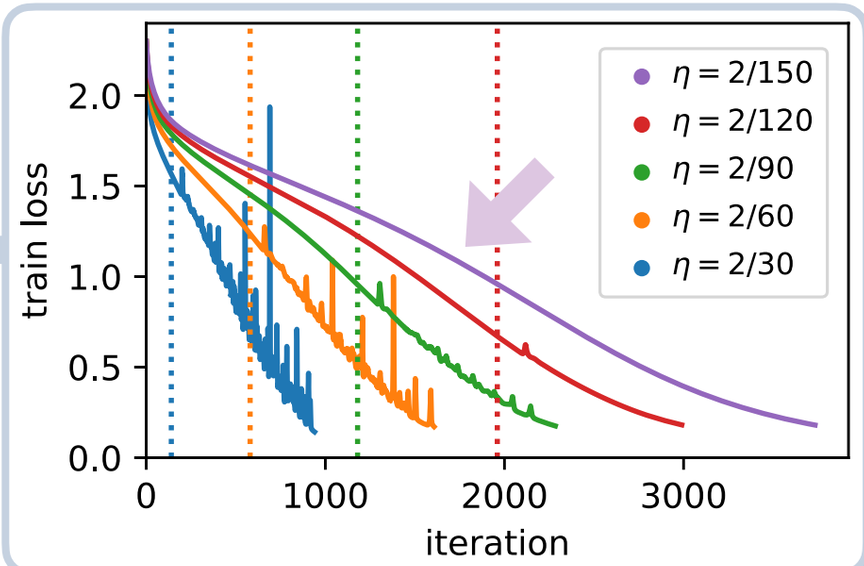
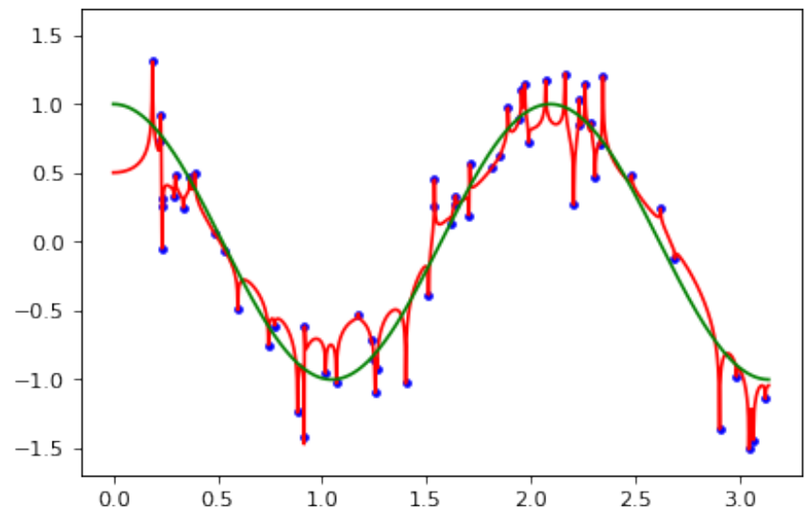



### generalization

overparameterization



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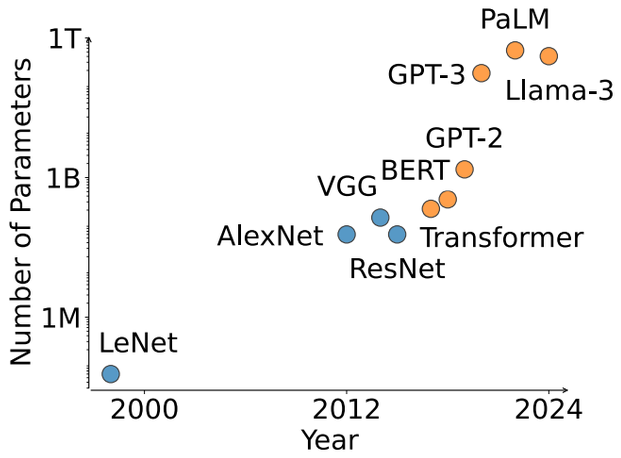


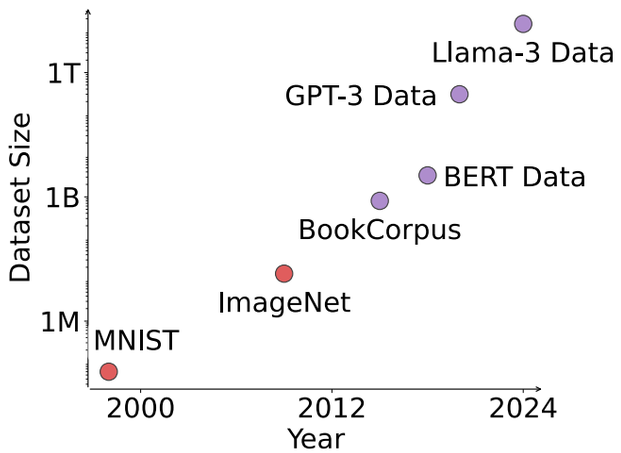


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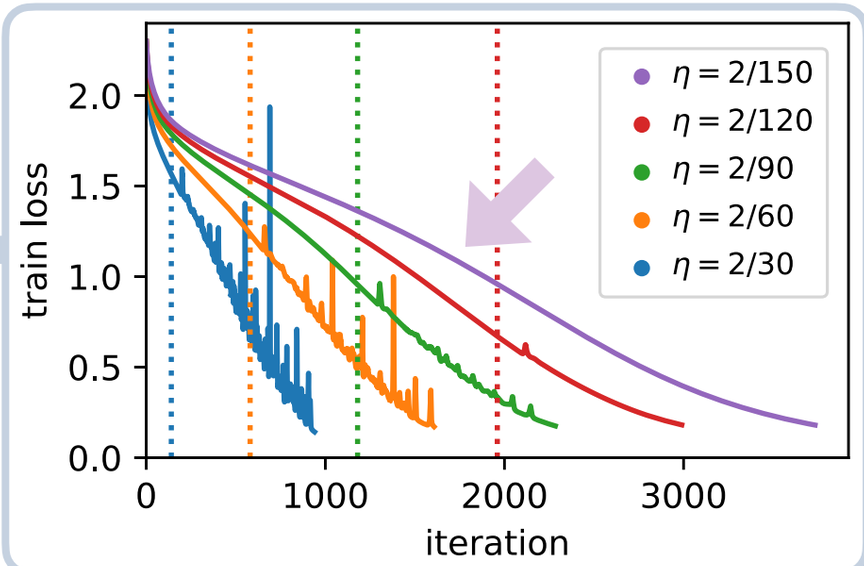


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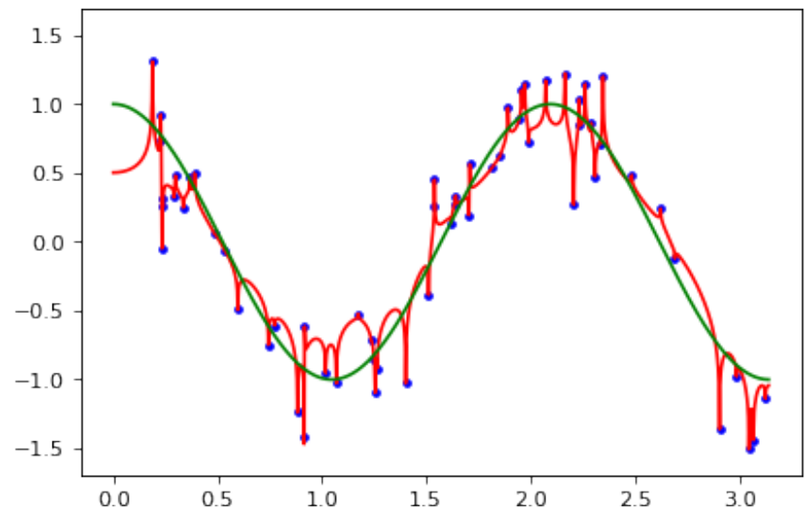



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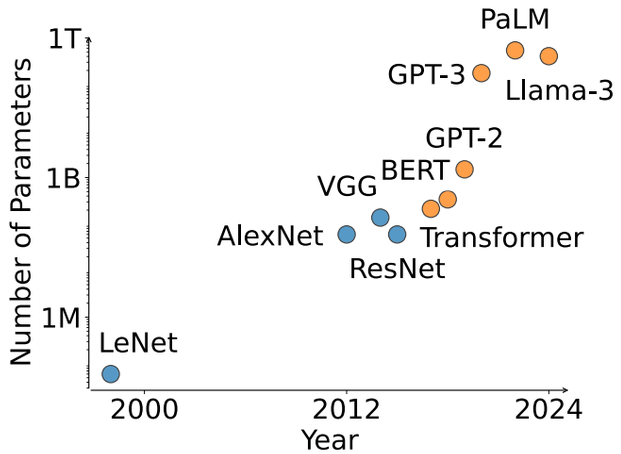


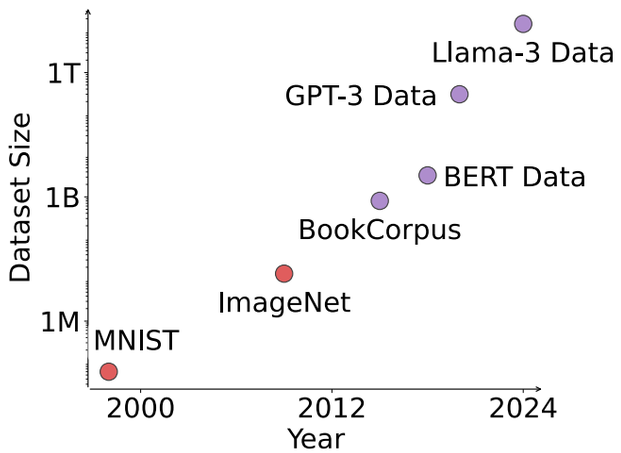


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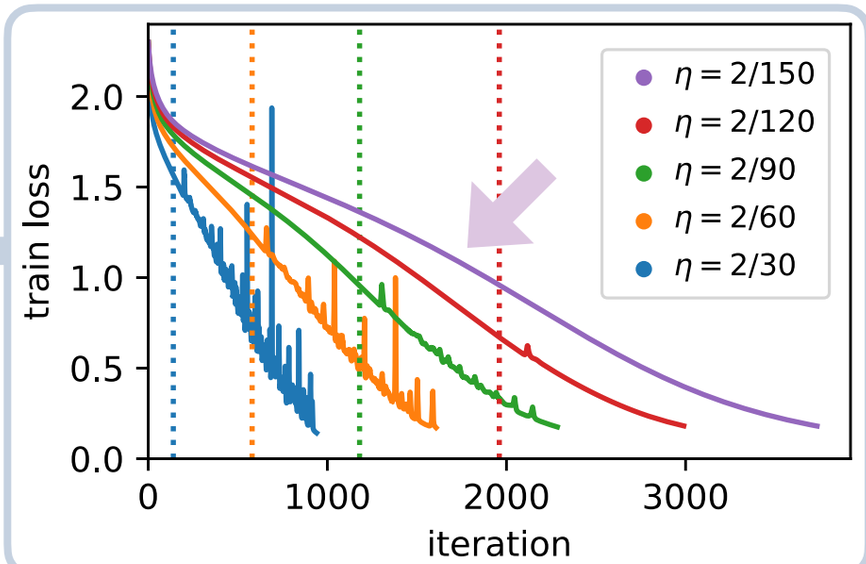


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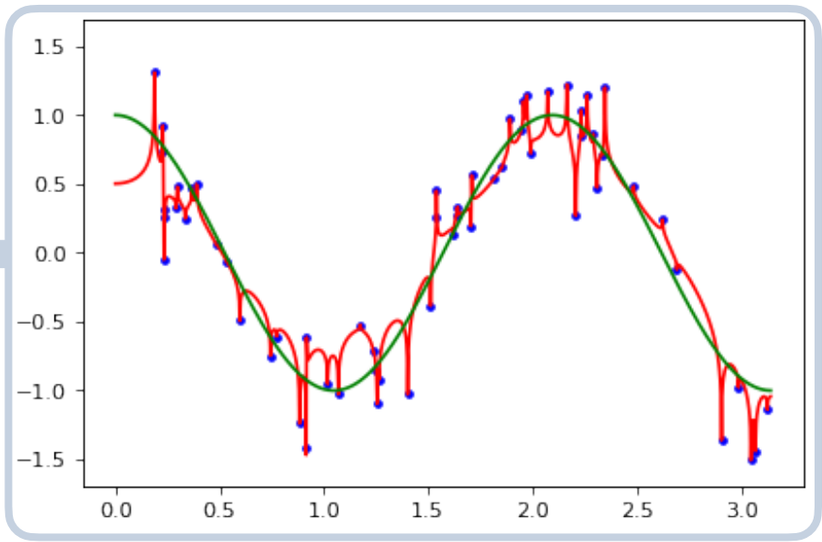



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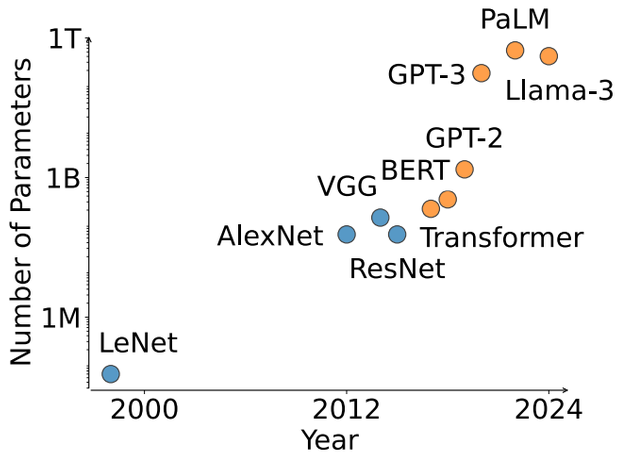


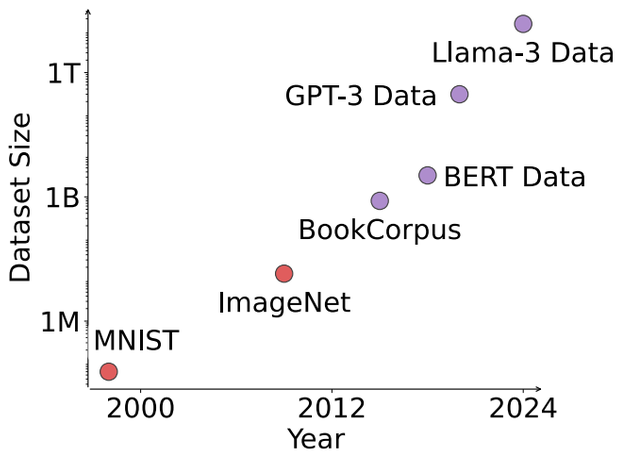


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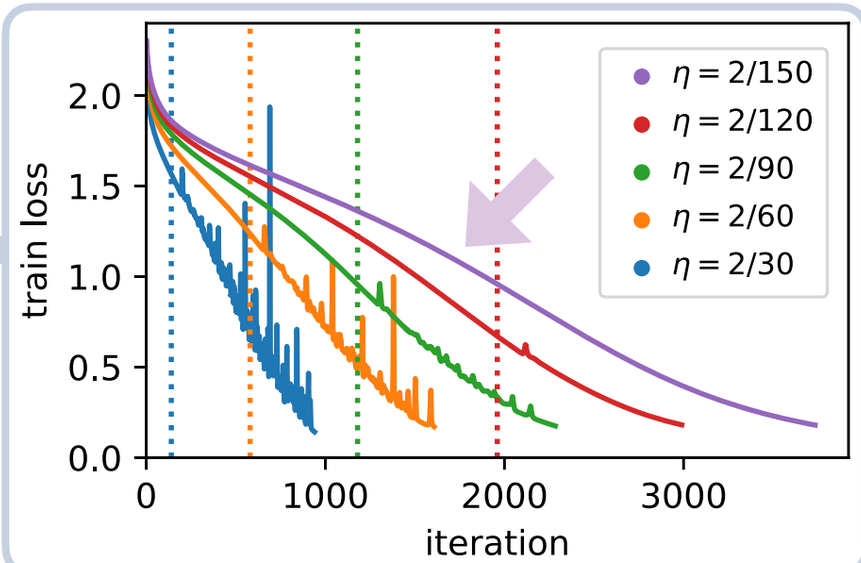


### generalization

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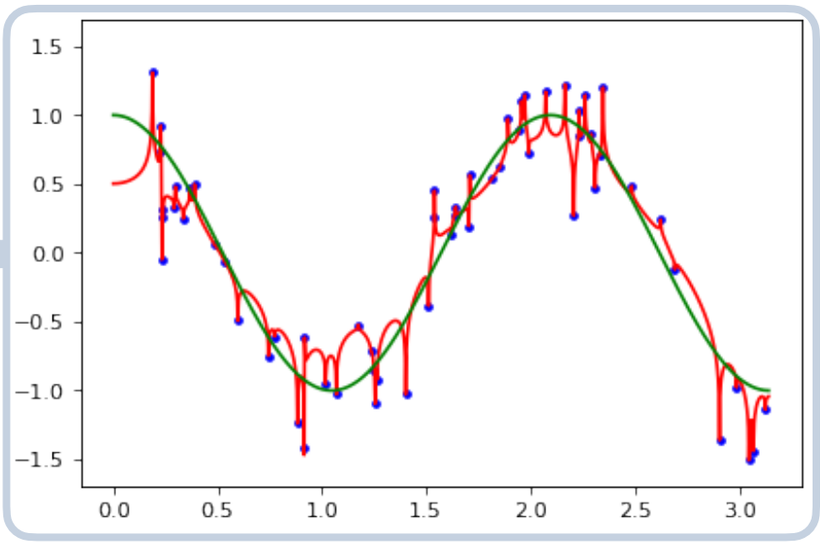
optimization path
**implicit regularization**





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# My research

deep learning = scaling + new mechanisms

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not covered by classical theory

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not specific to deep learning

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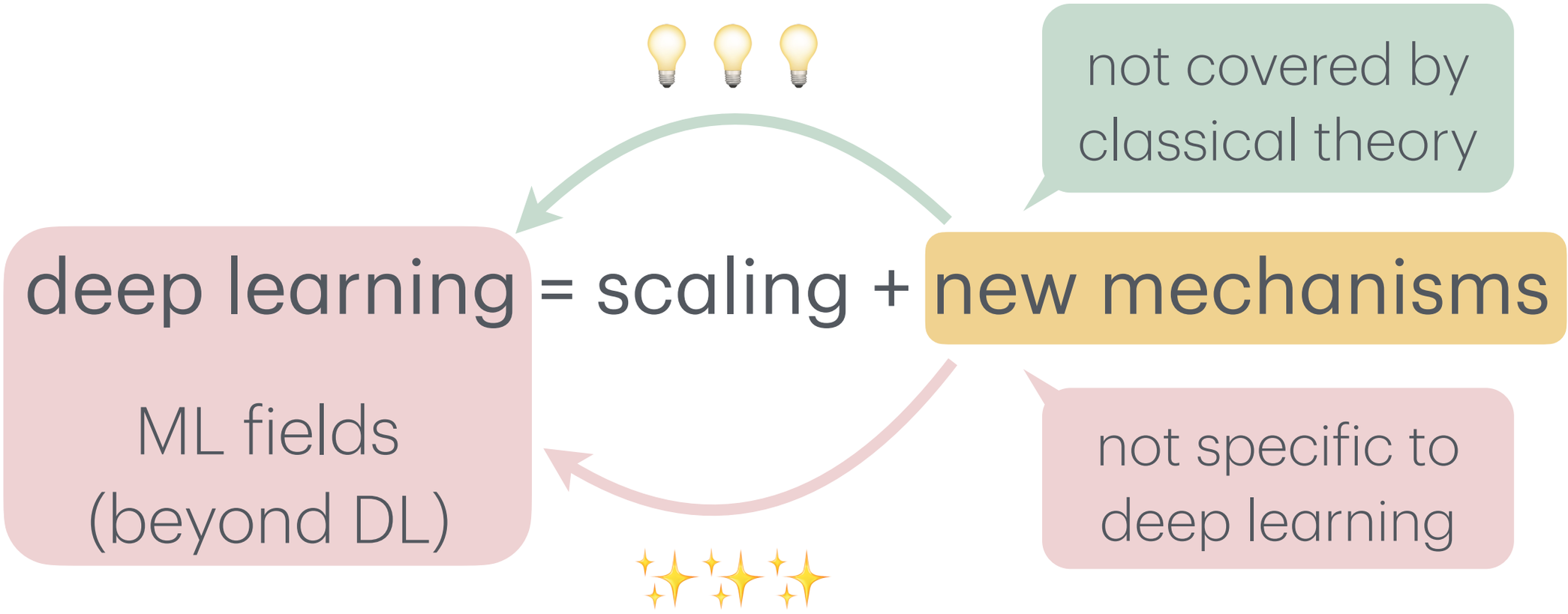
not covered by classical theory
deep learning = scaling + new mechanisms
ML fields (beyond DL)
not specific to deep learning

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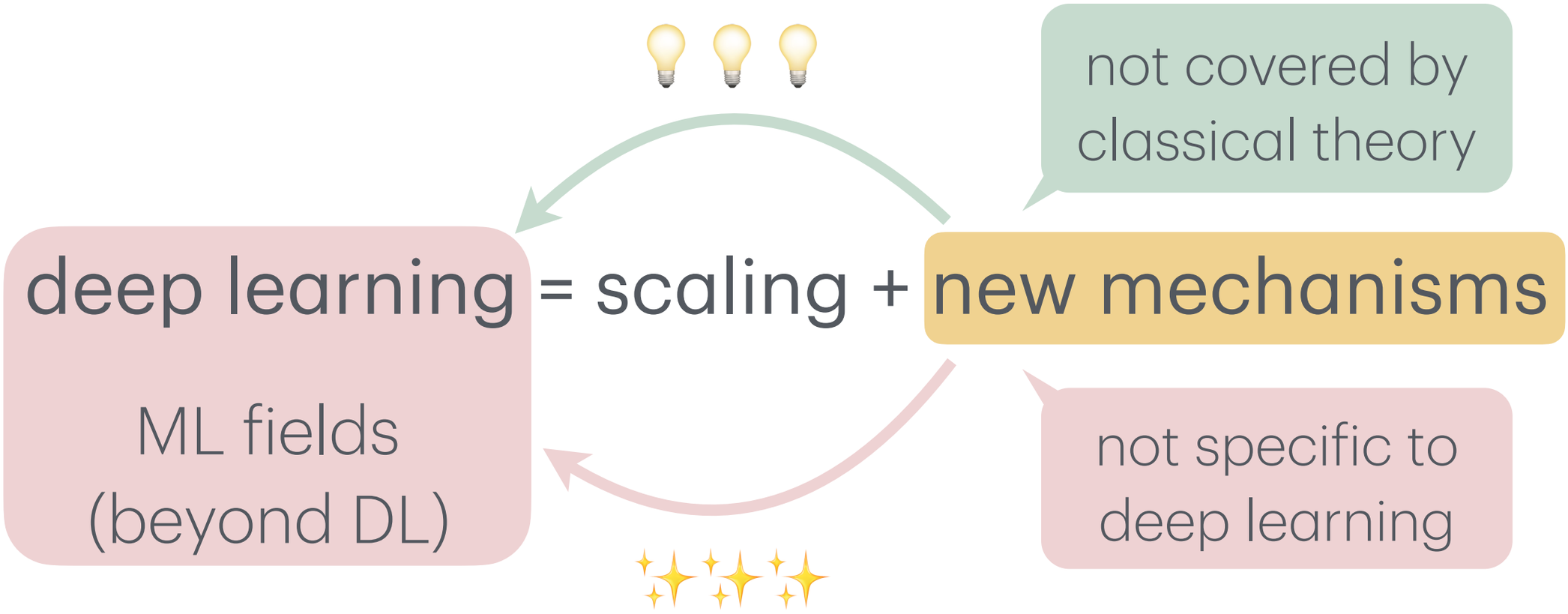


**Approach.** Demystify new mechanisms in **sandboxes**

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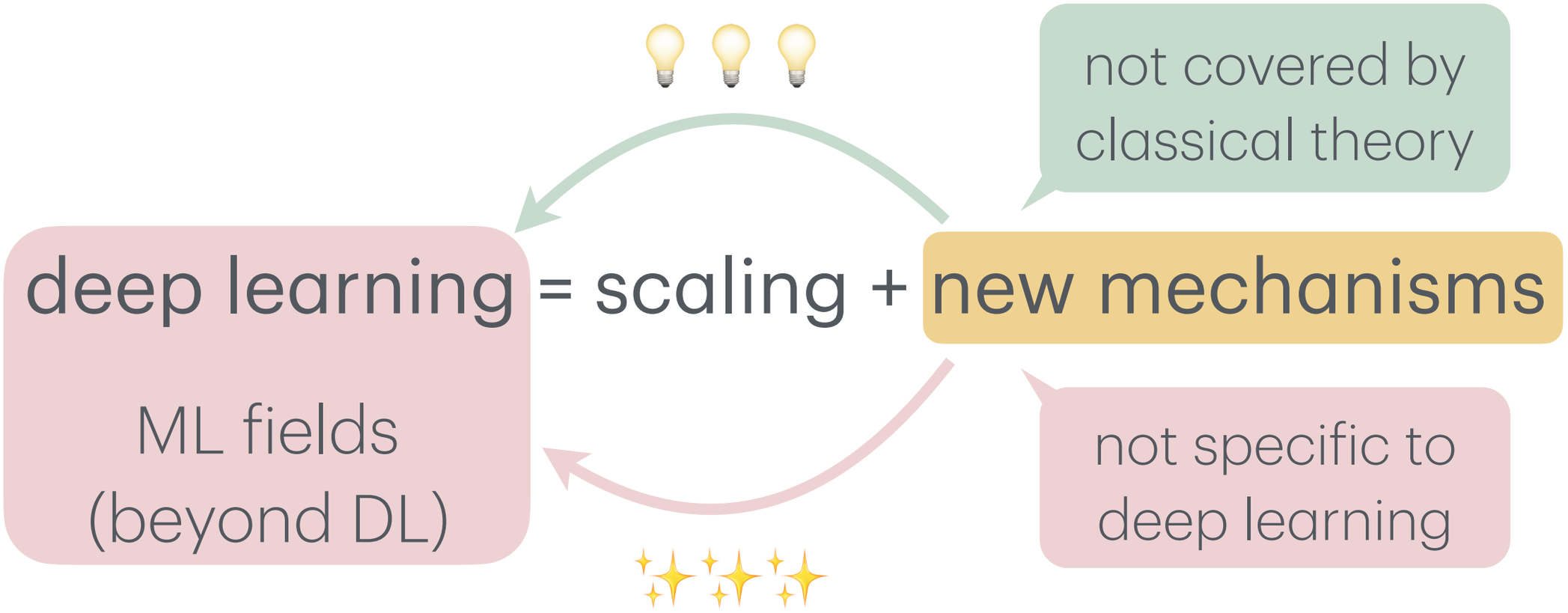


**Approach.** Demystify new mechanisms in **sandboxes**

simple

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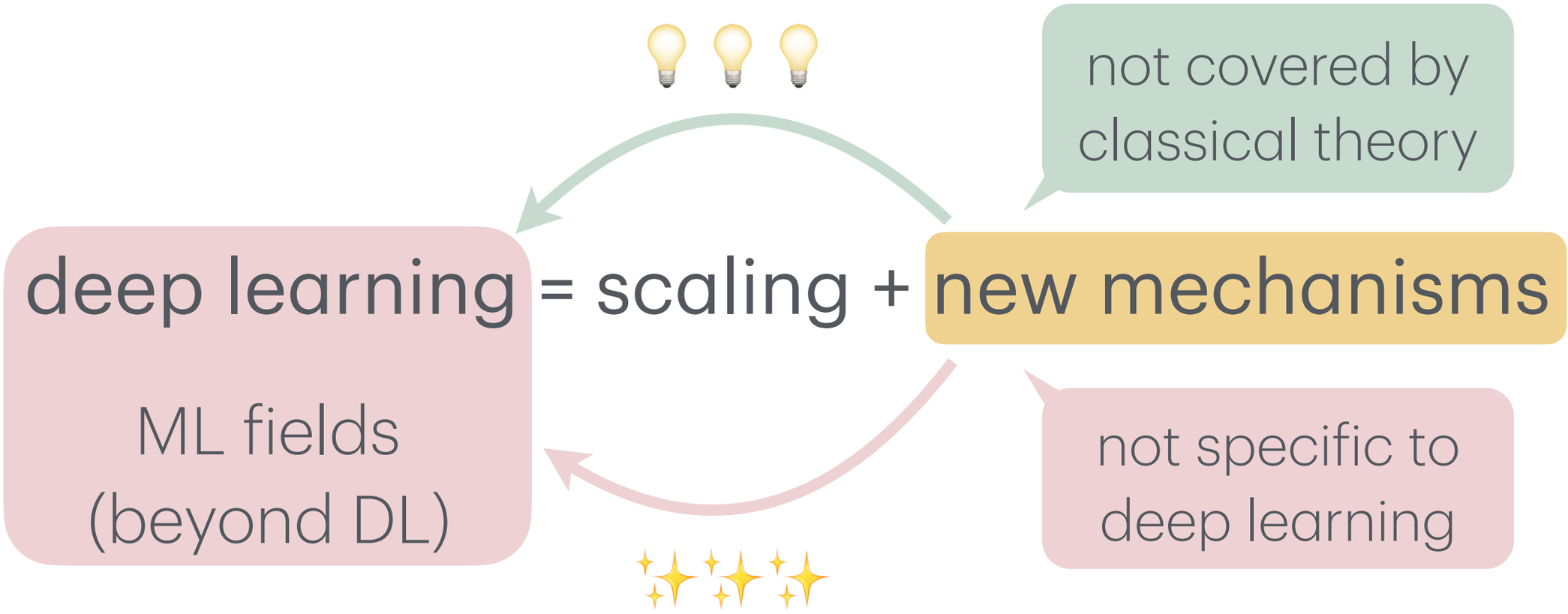


**Approach.** Demystify new mechanisms in **sandboxes**

simple

meaningful

## Contribution 1: unstable optimization

large stepsize accelerates gradient descent in logistic regression

## Contribution 2: implicit regularization

gradient descent dominates ridge regression in linear regression

## Contribution 3: from theory to practice

principled parallelization method for training language models

# Contribution 1: unstable optimization

large stepsize accelerates gradient descent in logistic regression

- “Large stepsize gradient descent for logistic loss: non-monotonicity of the loss improves optimization efficiency”
  **W**, Peter Bartlett, Matus Telgarsky, Bin Yu
  COLT 2024

- “Large stepsizes accelerate gradient descent for regularized logistic regression”
  **W***, Pierre Marion*, Peter Bartlett
  NeurIPS 2025

# Unstable optimization

Gradient Descent $\theta_{t+1} = \theta_t - \eta \nabla L(\theta_t)$

# Unstable optimization

how to choose $\eta$?

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**classical theory:**
small stepsize for stability
$L(\theta_t) \downarrow$

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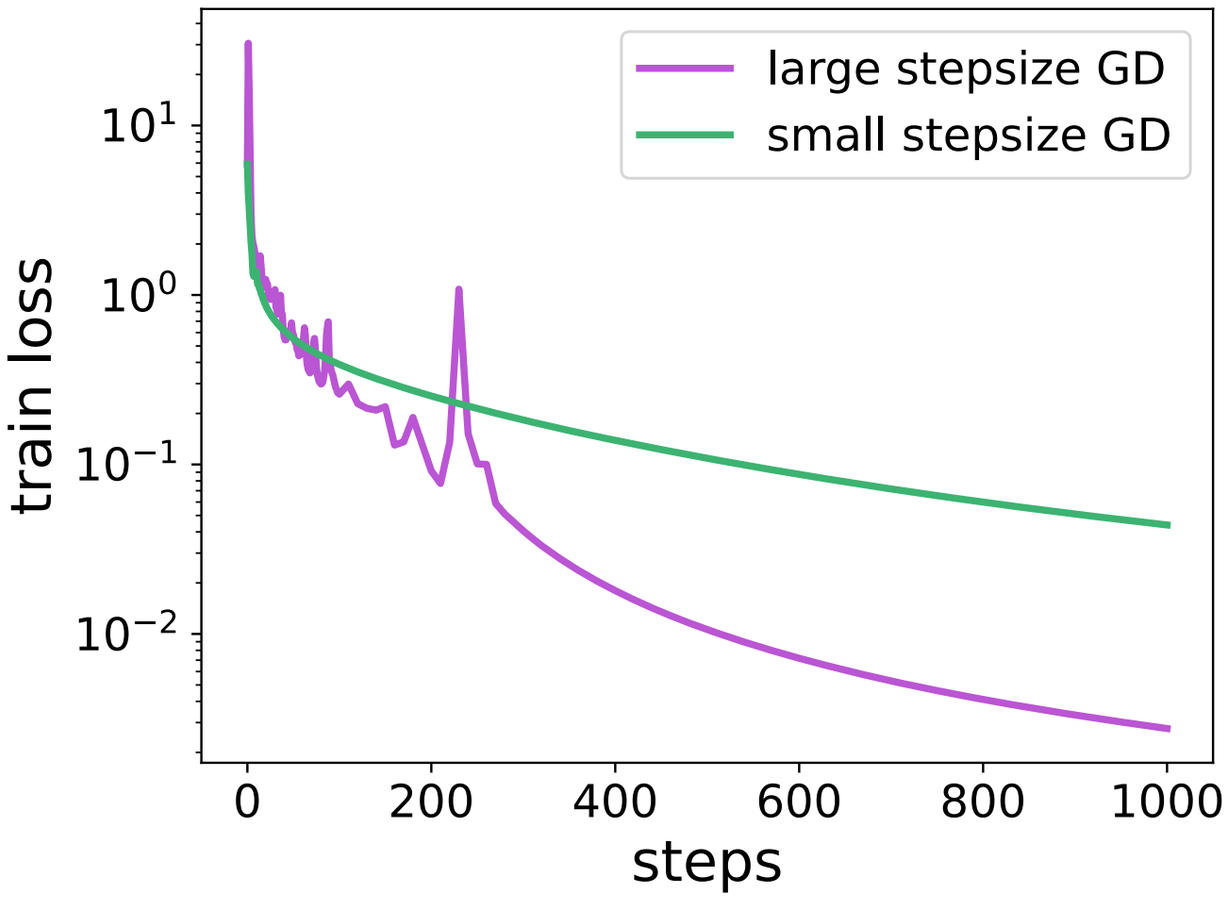


MLP, GD, classification task

**classical theory:**
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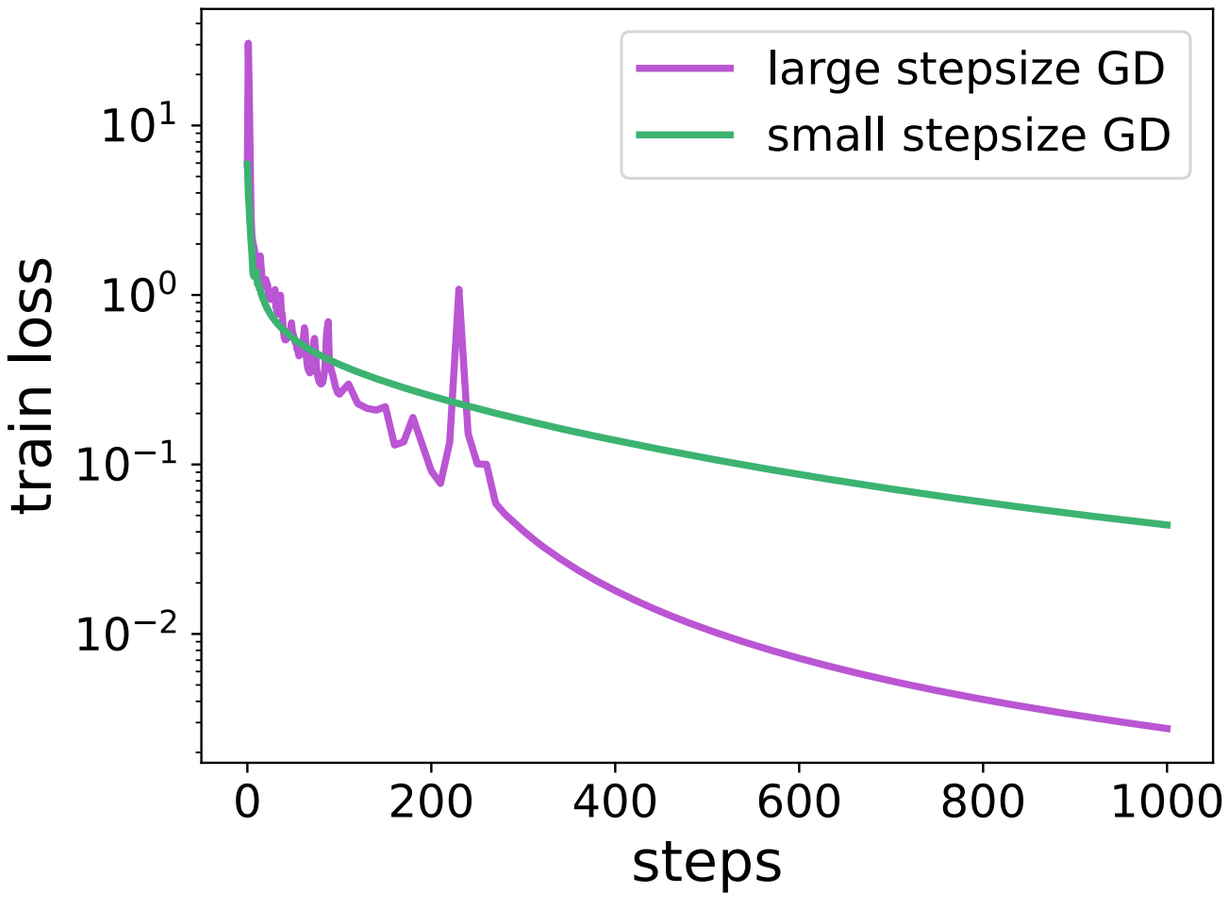


MLP, GD, classification task

**classical theory:**
small stepsize for stability
$L(\theta_t) \downarrow$

**practice:**
best stepsize is large
oscillatory loss

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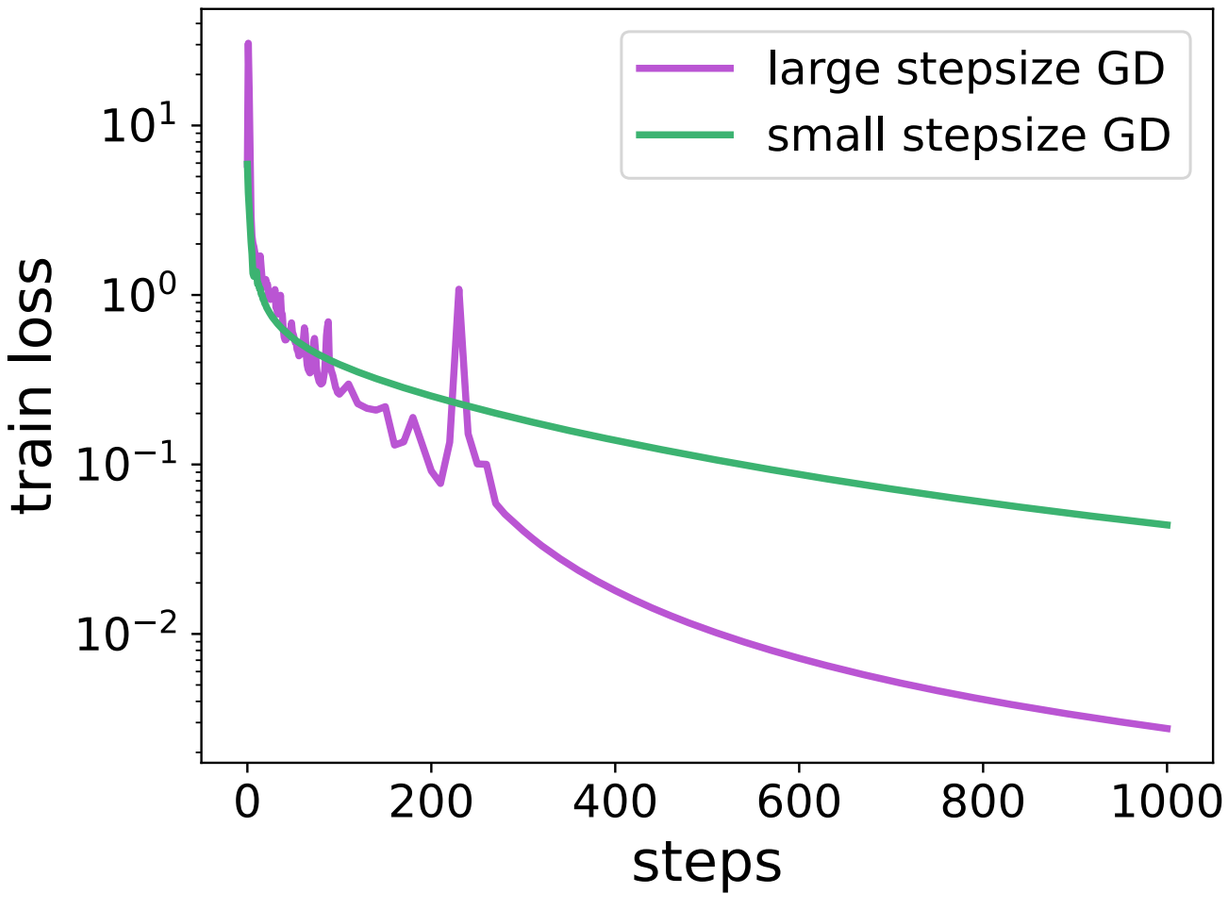


MLP, GD, classification task

**classical theory:**
small stepsize for stability
$L(\theta_t) \downarrow$

**practice:**
best stepsize is large
oscillatory loss

classical theory fails to predict best stepsize


Cohen, Kaur, Li, Kolter, Talwalkar. "Gradient descent on neural networks typically occurs at the edge of stability." ICLR 2021
**Wu**, Wang, Fazel. "Theoretical insights on training instability in deep learning." Tutorial@NeurIPS 2025

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Let $L$ be 1-smooth ($\|\nabla^2 L\| \leq 1$) with finite minimizer

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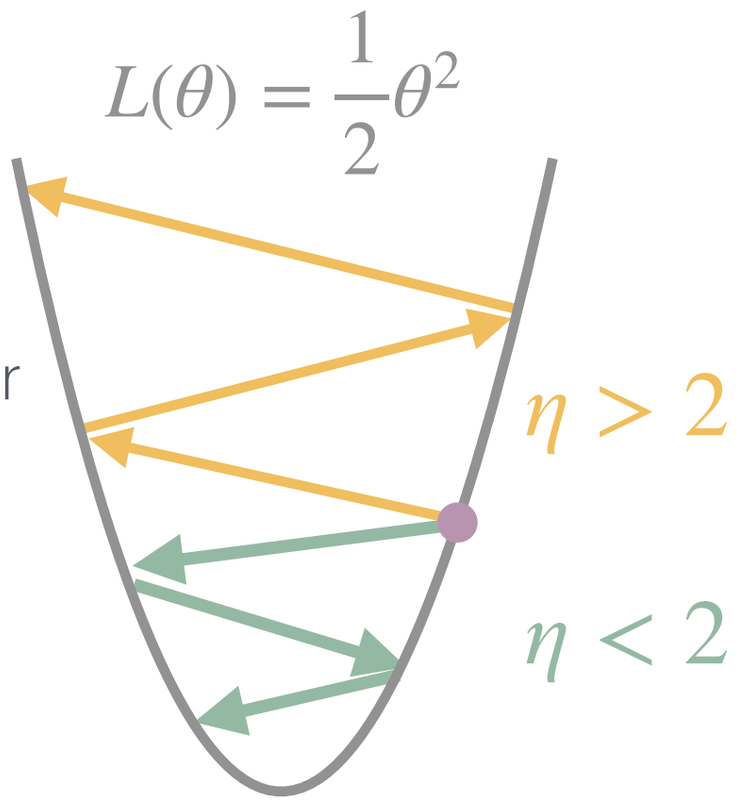


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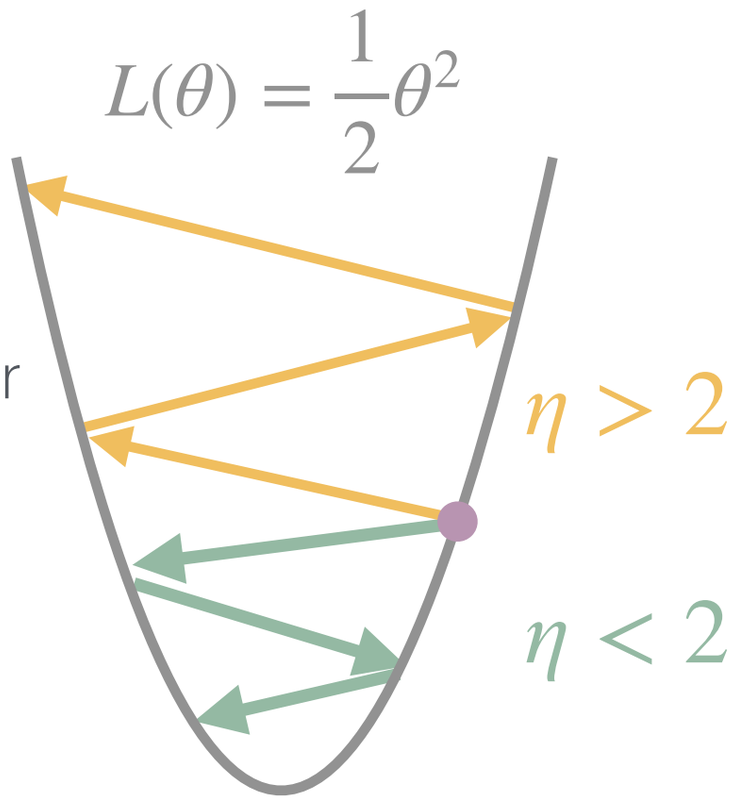


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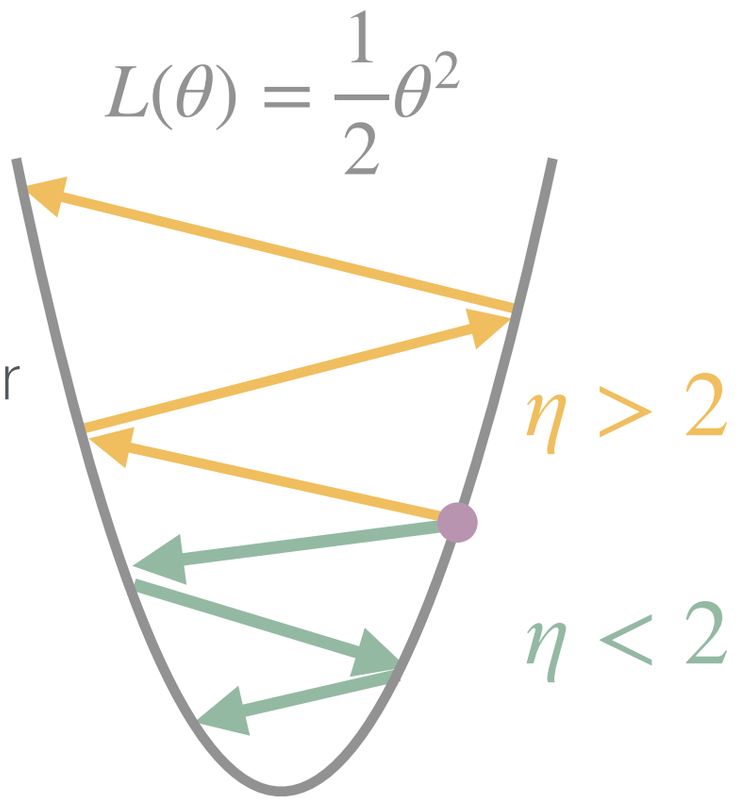


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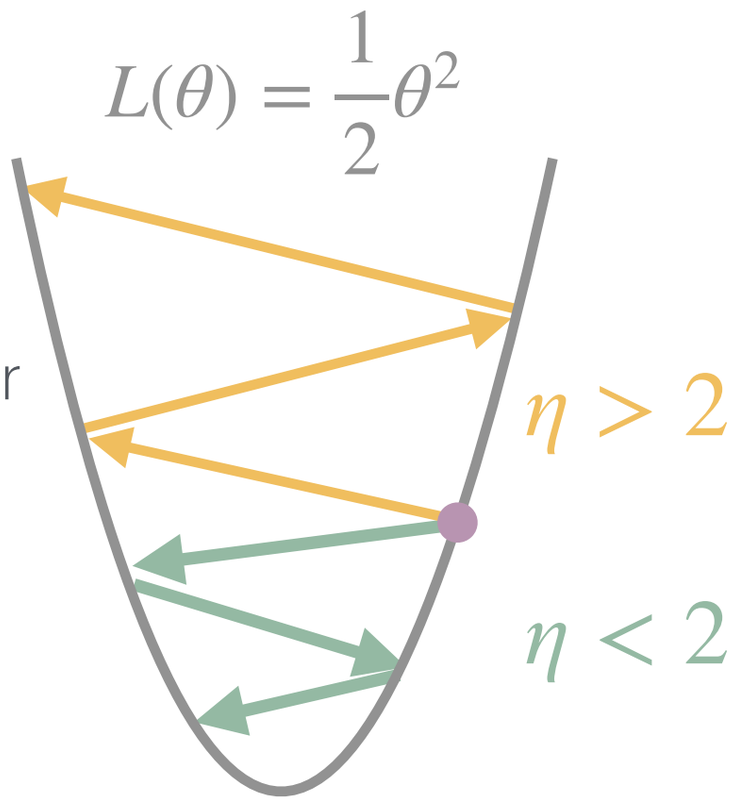


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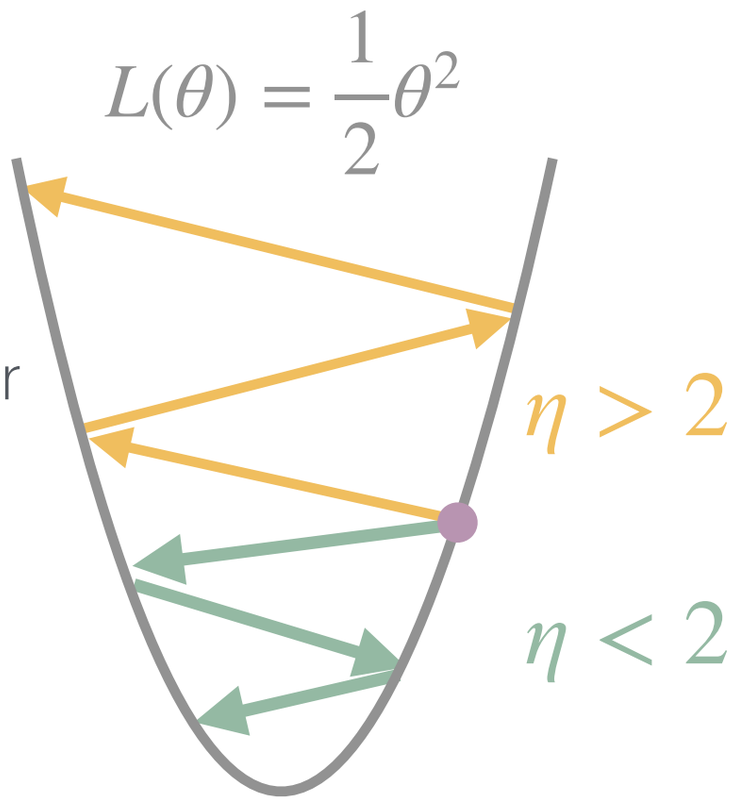


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acceleration by Nesterov's momentum: $O\big(1/t^2\big)$ & $O\big(\sqrt{\kappa}\ln(1/\epsilon)\big)$

# Stepsize

$\eta = \infty$

divergence
(in worst cases)

$\eta = o(1)$

classical theory
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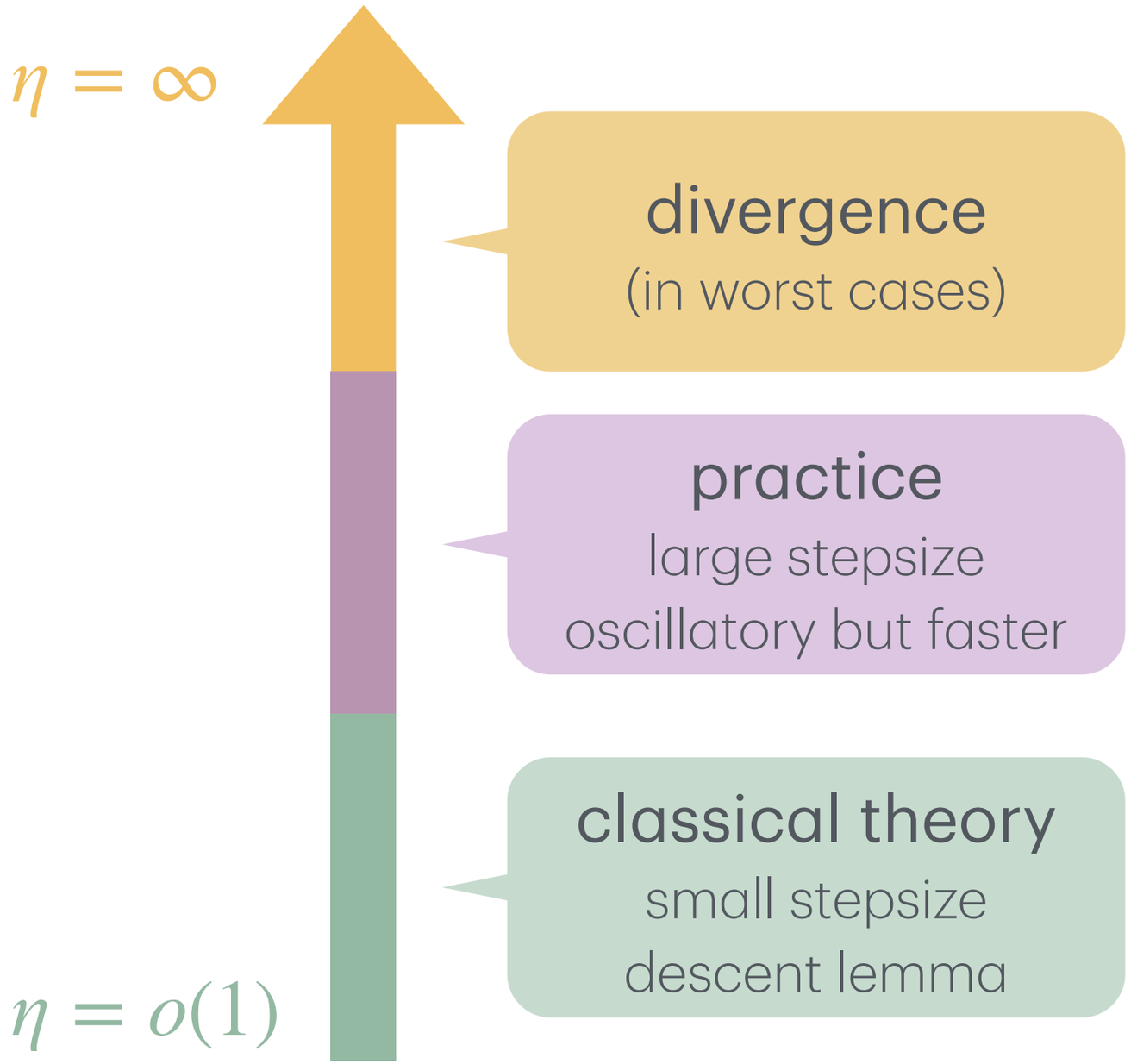
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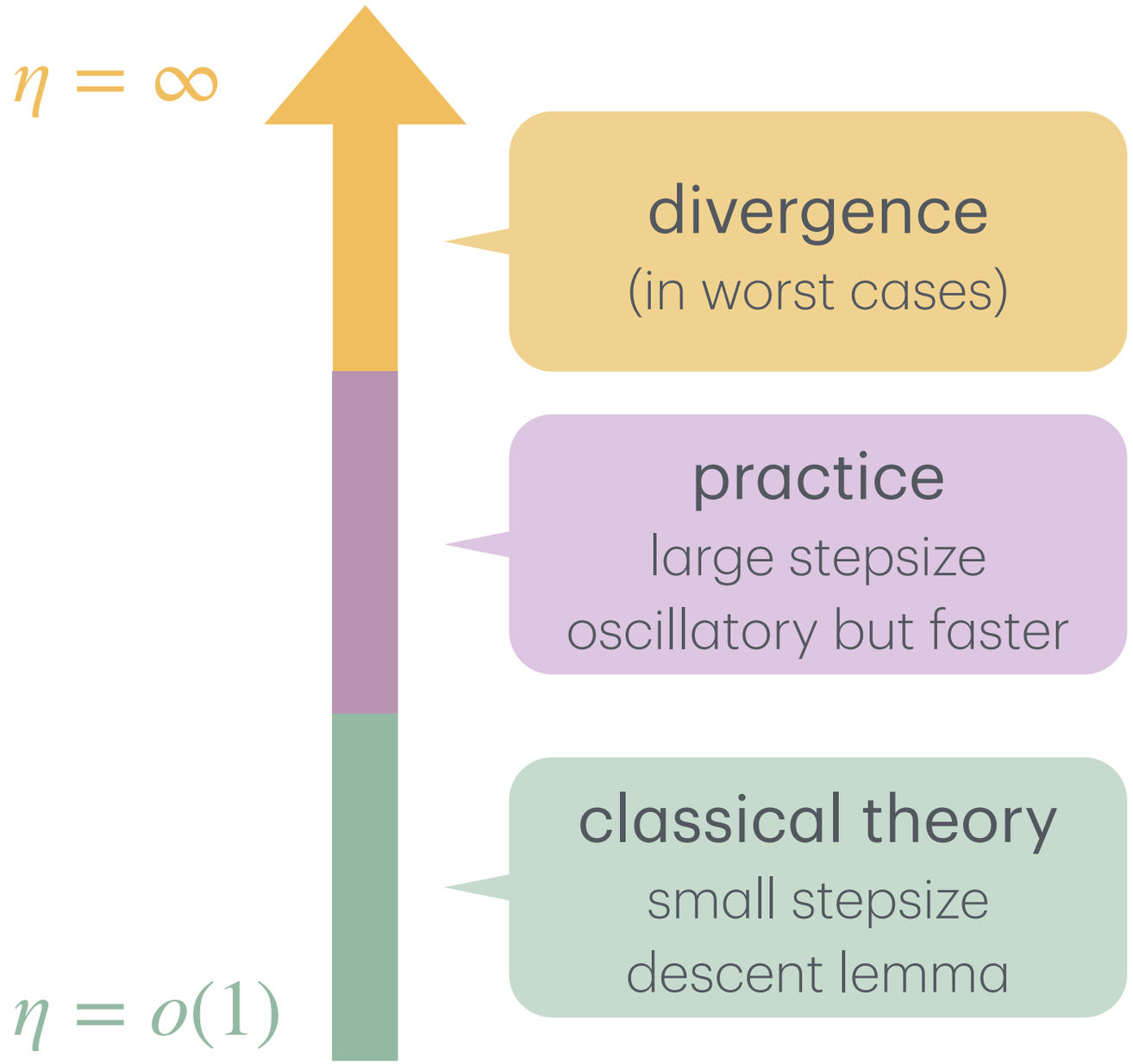
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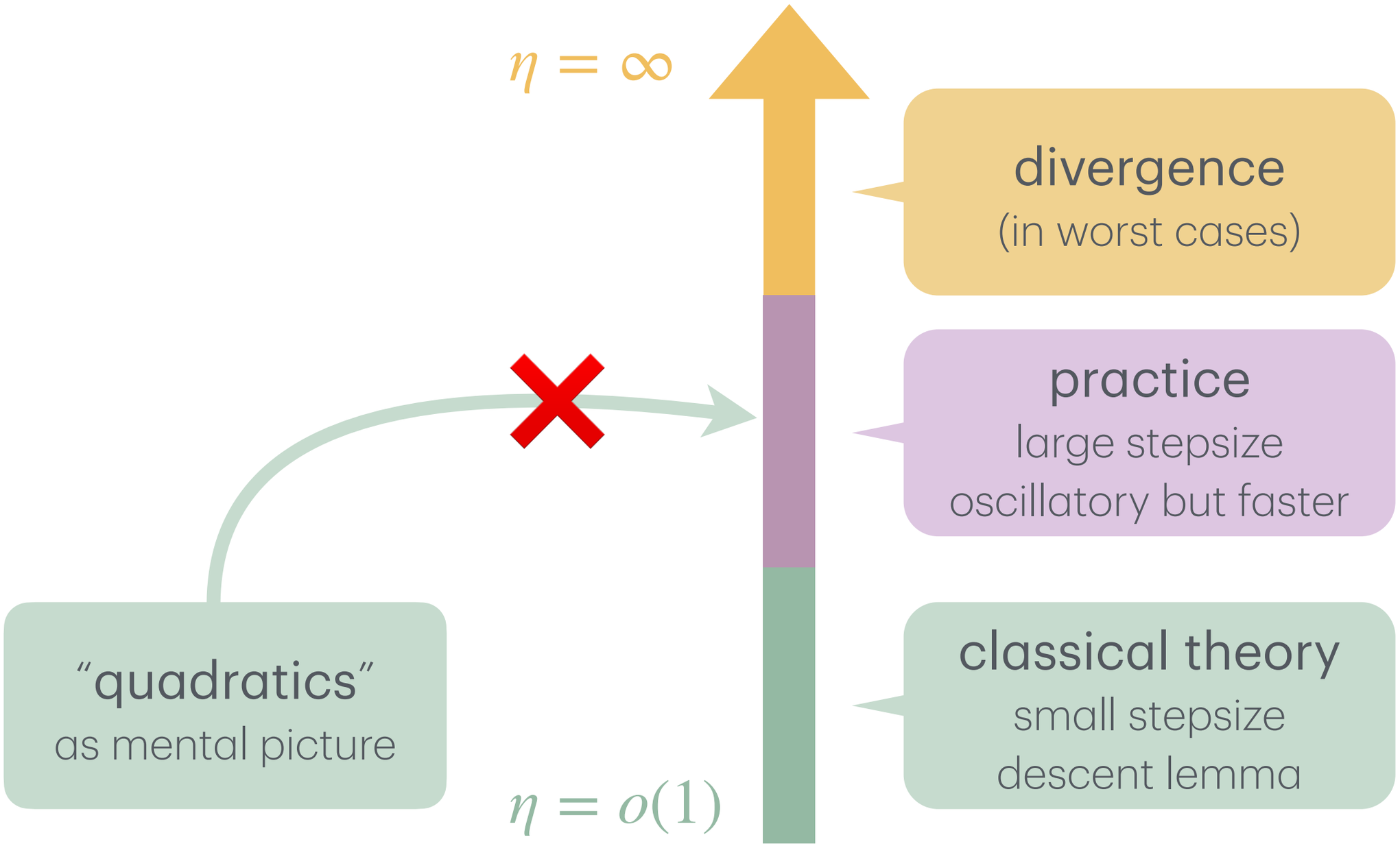
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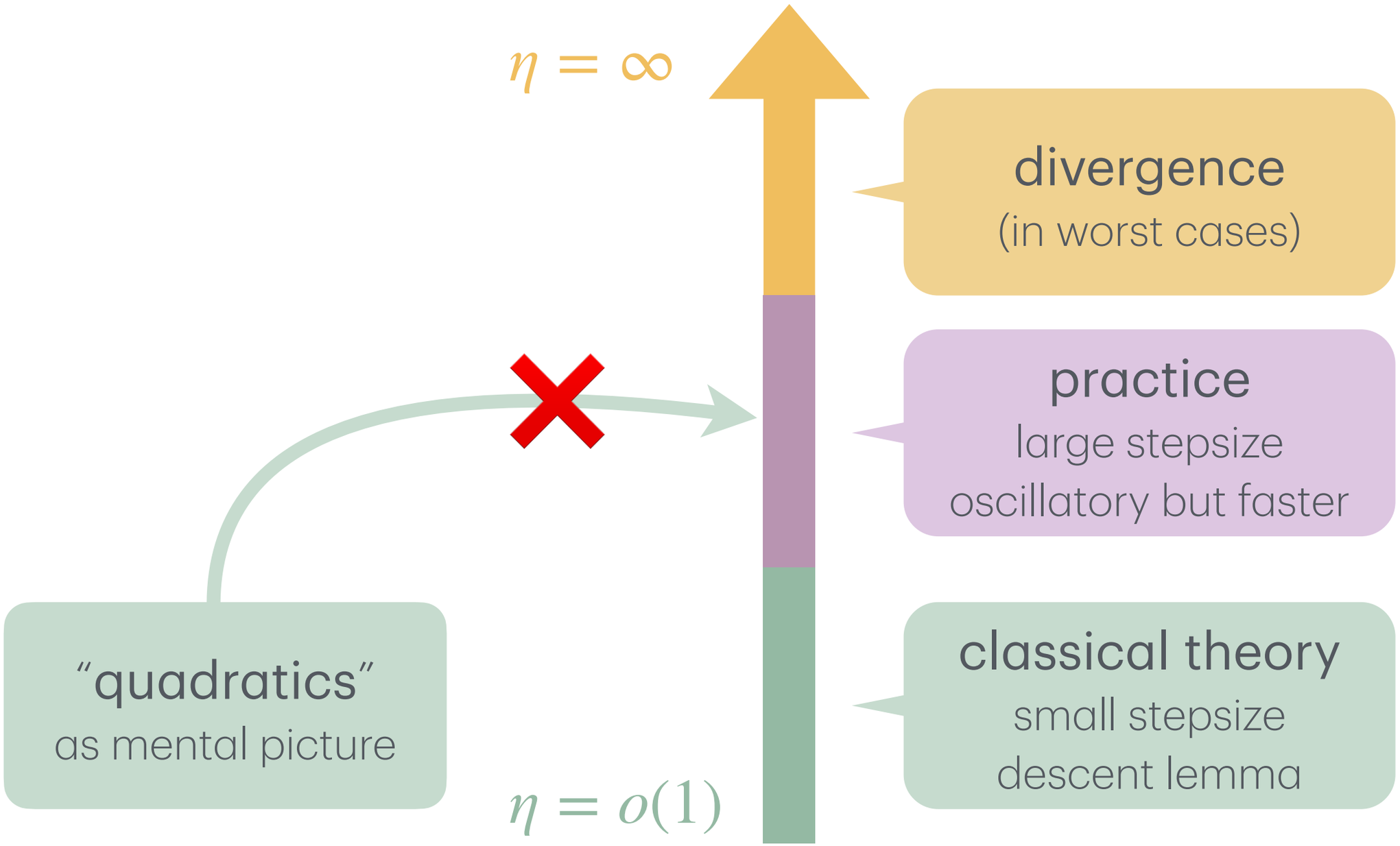


Related works

- Altschuler, Parrilo. "Acceleration by stepsize hedging I: multi-step descent and the silver stepsize schedule." Journal of the ACM 2024
- Grimmer, Shu, Wang. "Composing optimized stepsize schedules for gradient descent." Mathematics of Operations Research 2025

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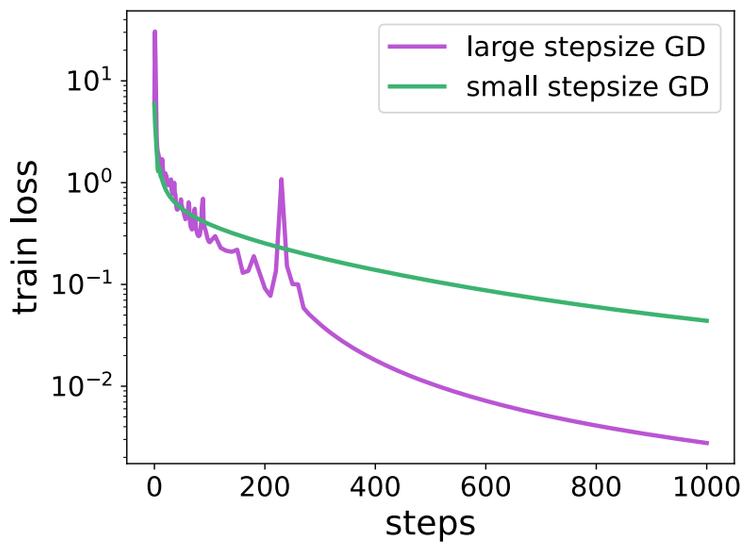
# Seeking simplest sandbox

# Seeking simplest sandbox

linear regression

.....

deep learning

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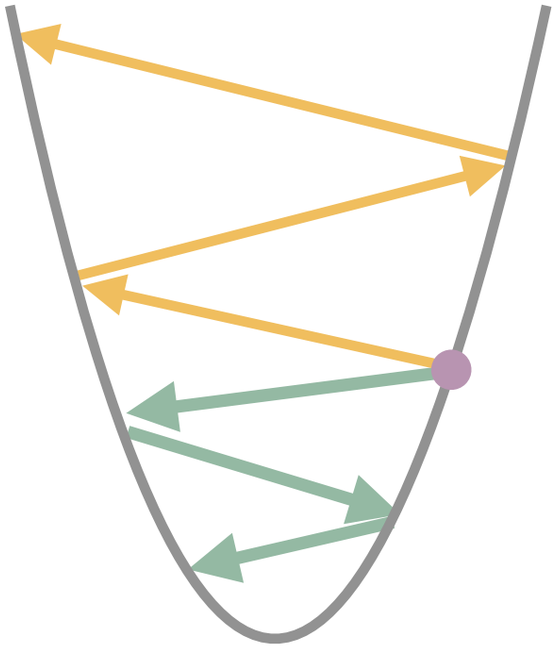
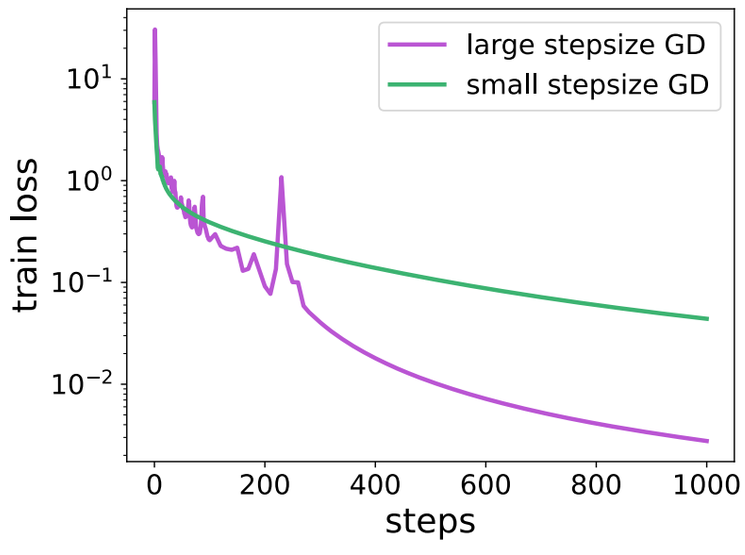
unstable optimization observed





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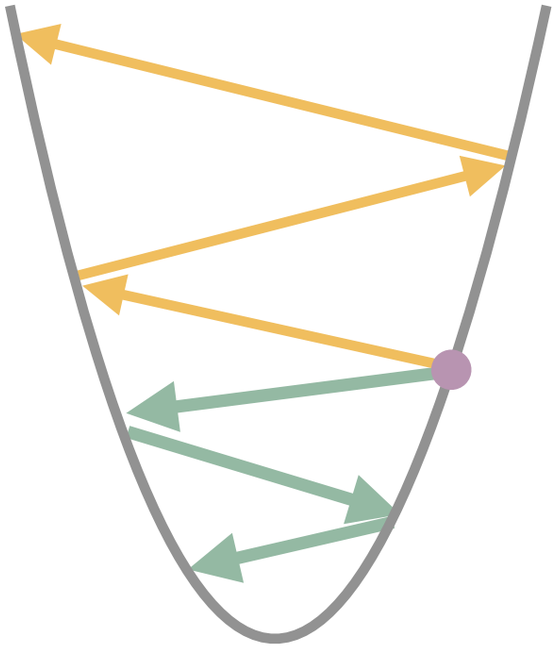

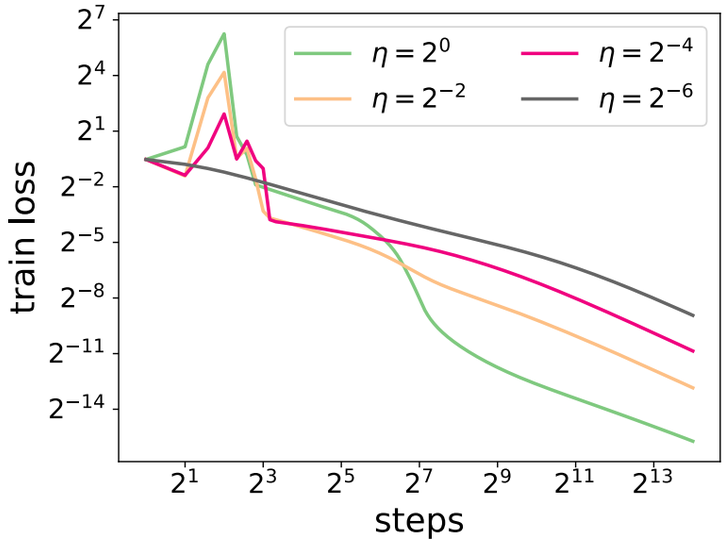


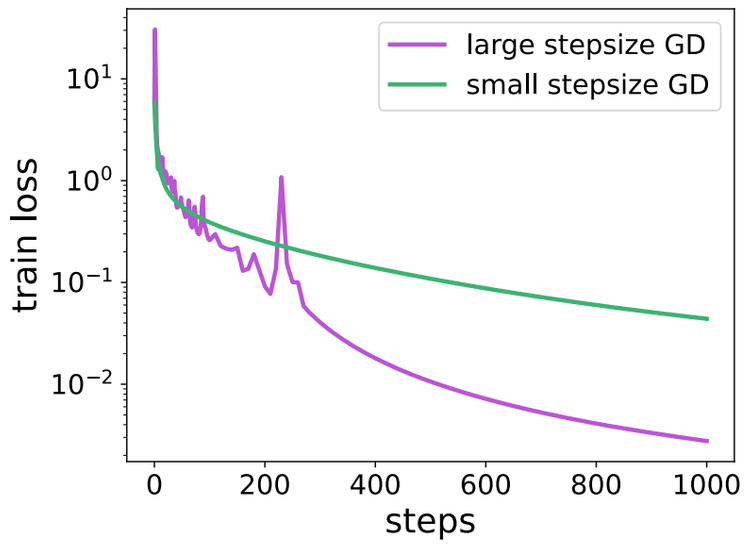

# Logistic regression

empirical risk $$L(\theta) = \frac{1}{n}\sum_{i=1}^{n} \ln\left(1 + \exp(-y_i x_i^\top \theta)\right)$$

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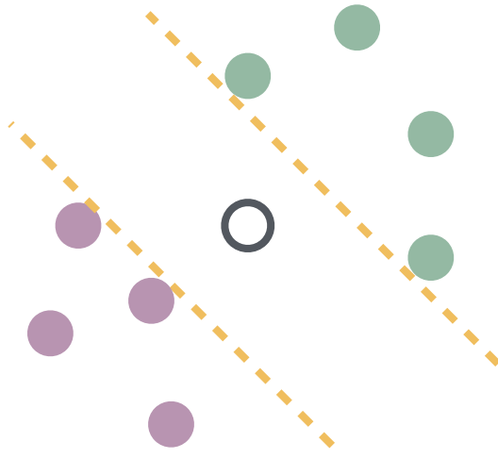

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smooth, convex
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improved to $\tilde{O}(1/t^2)$ by Nesterov

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**Wu**, Bartlett, Telgarsky, Yu. "Large stepsize gradient descent for logistic loss: non-monotonicity of the loss improves optimization efficiency." COLT 2024

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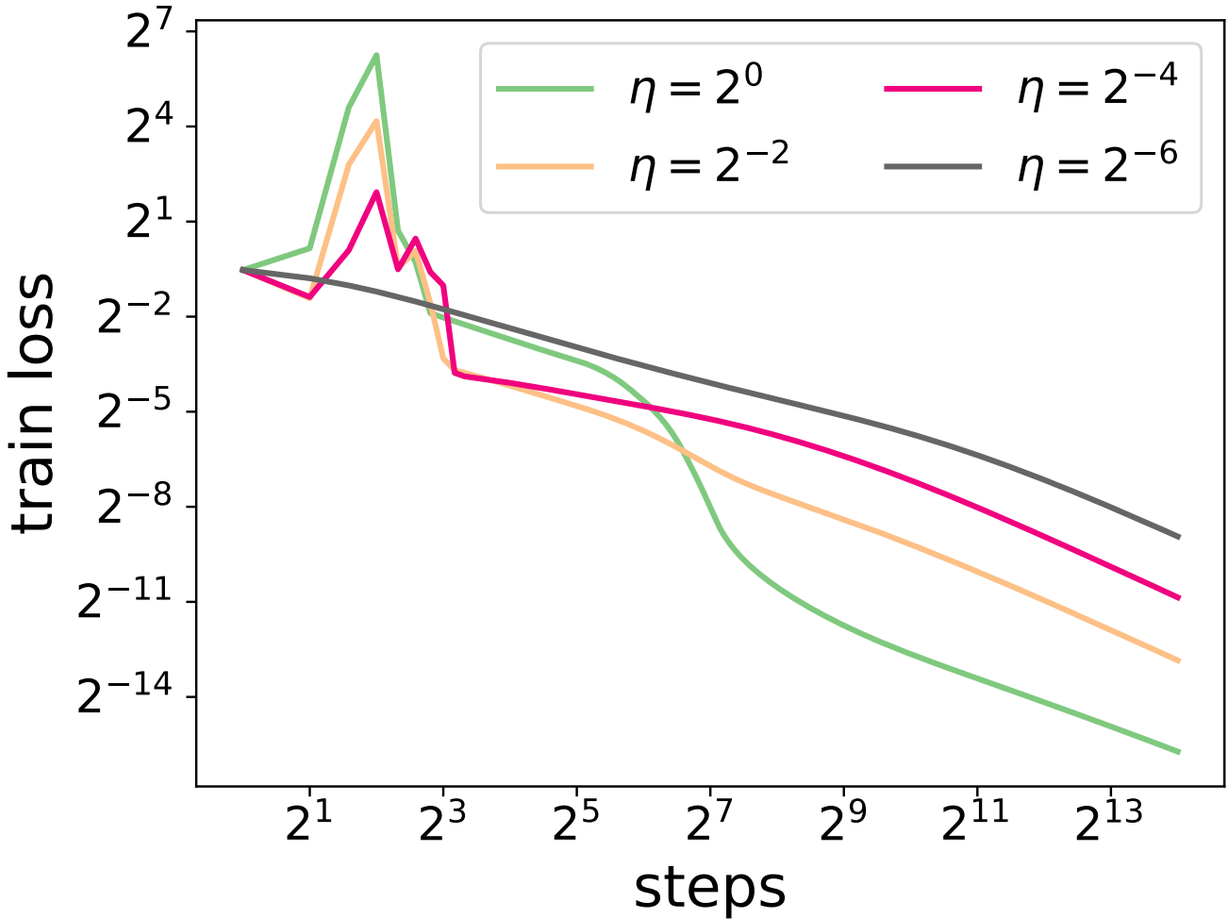


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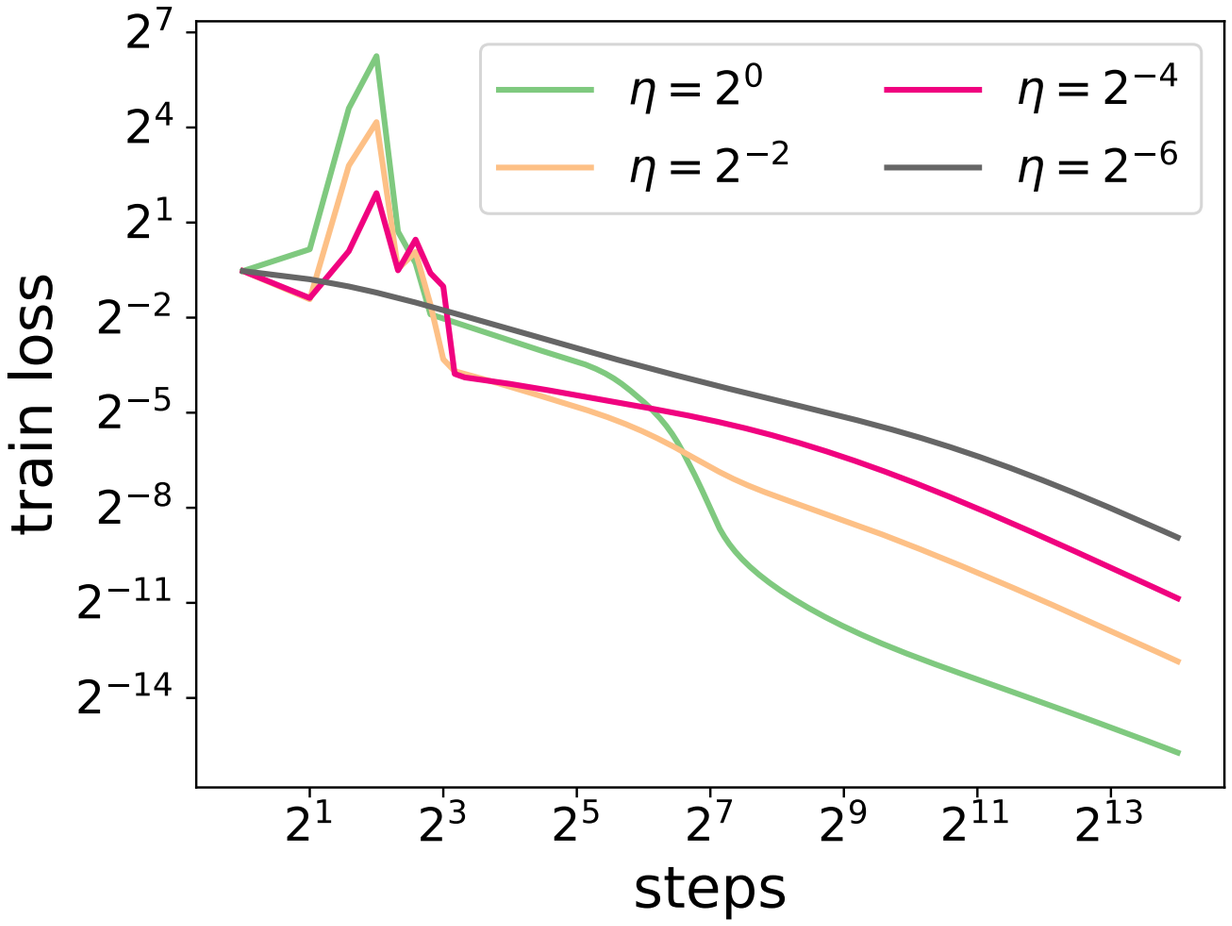


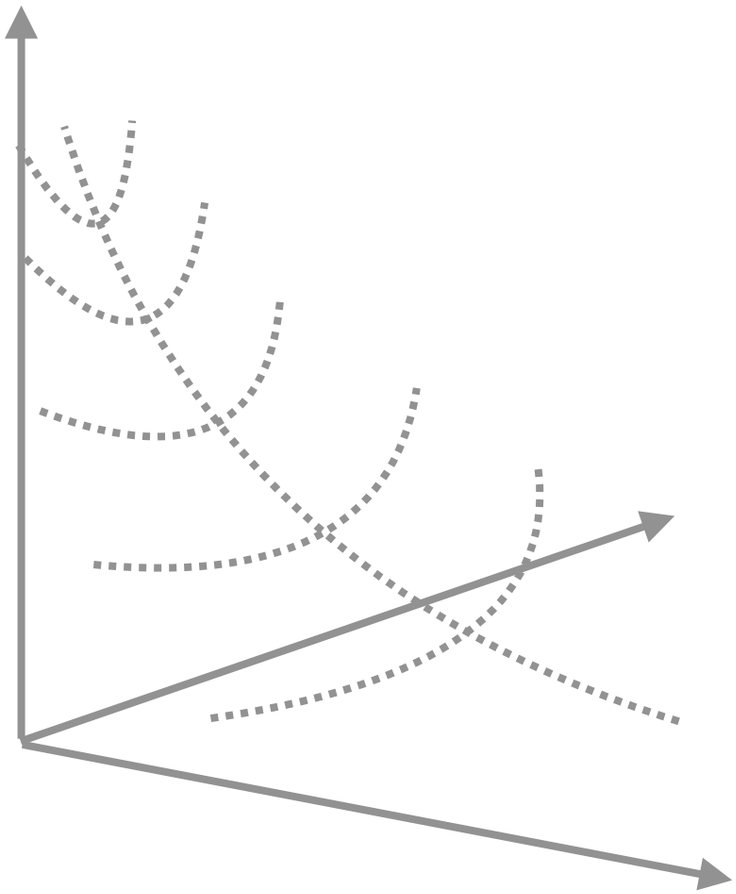

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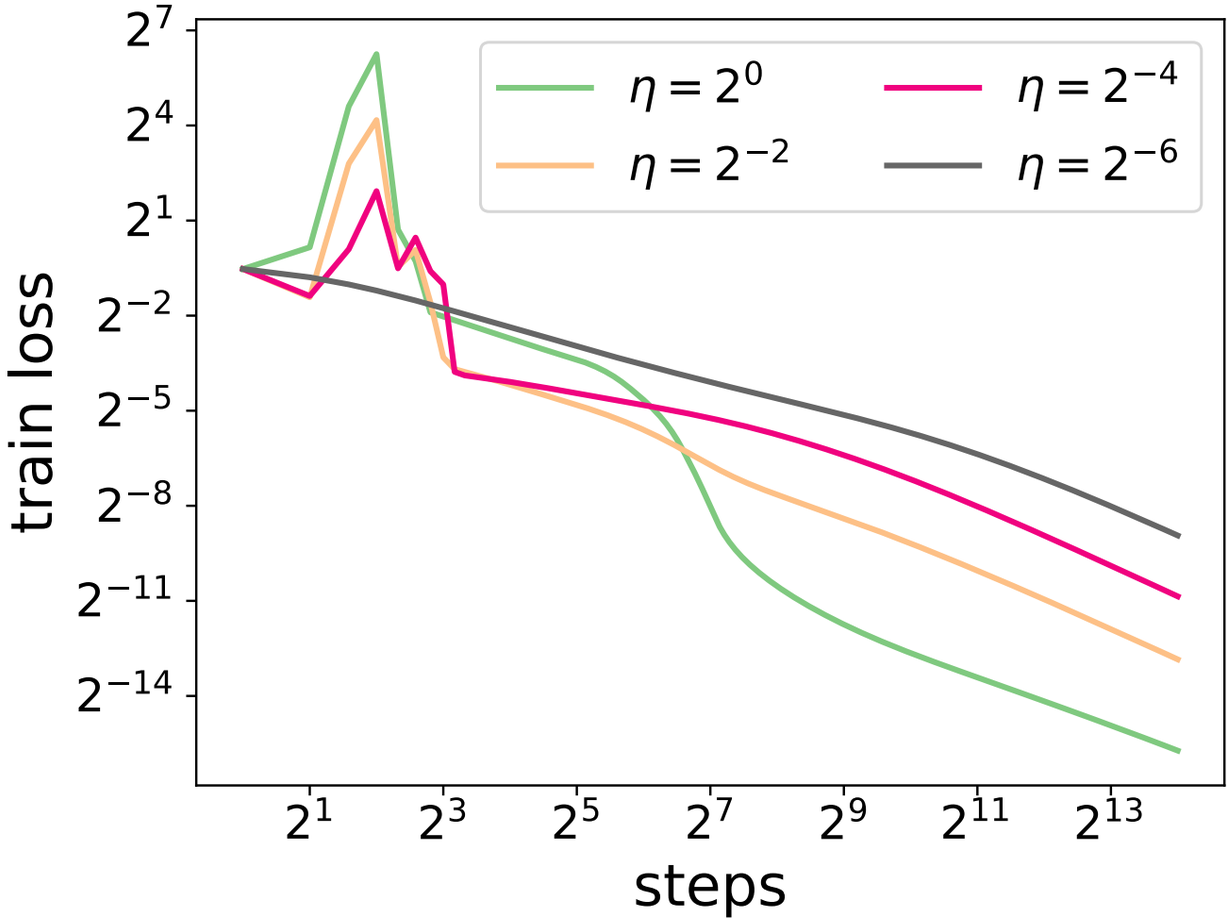


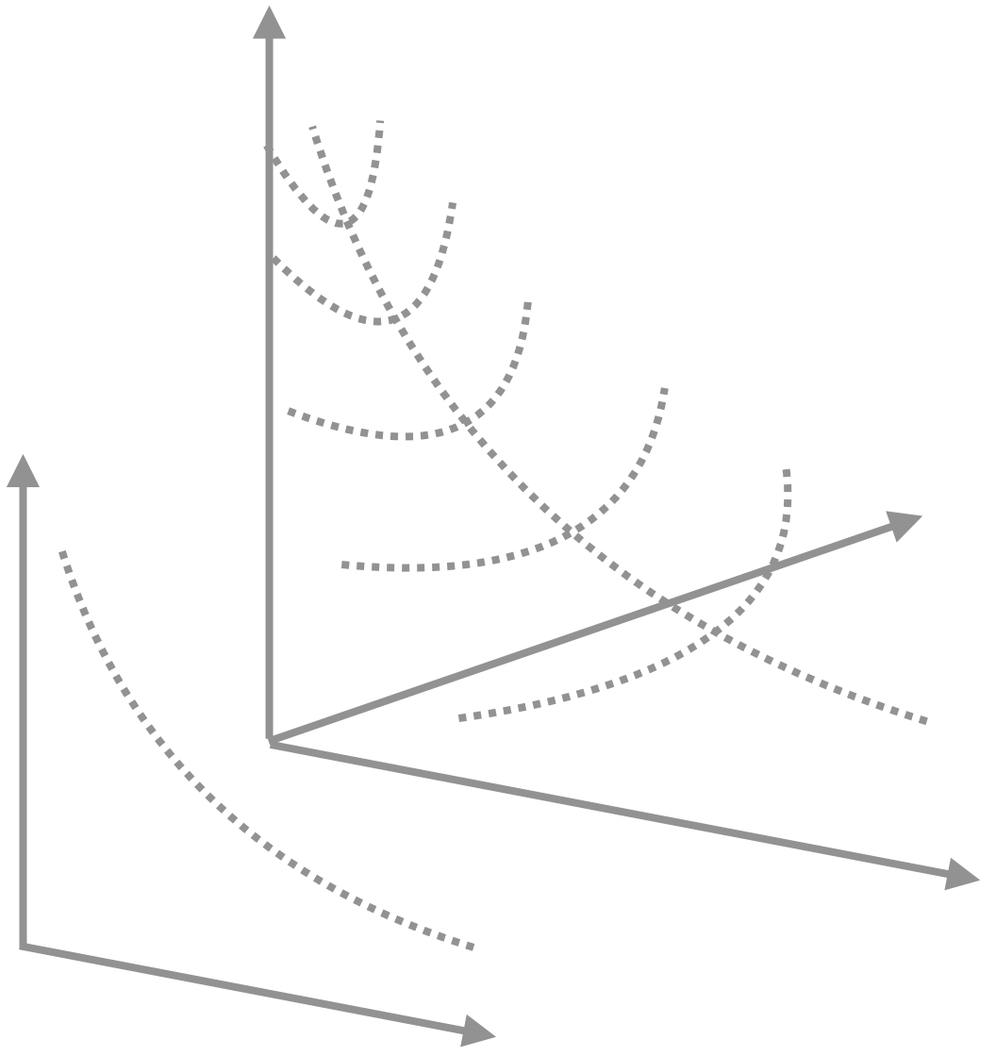

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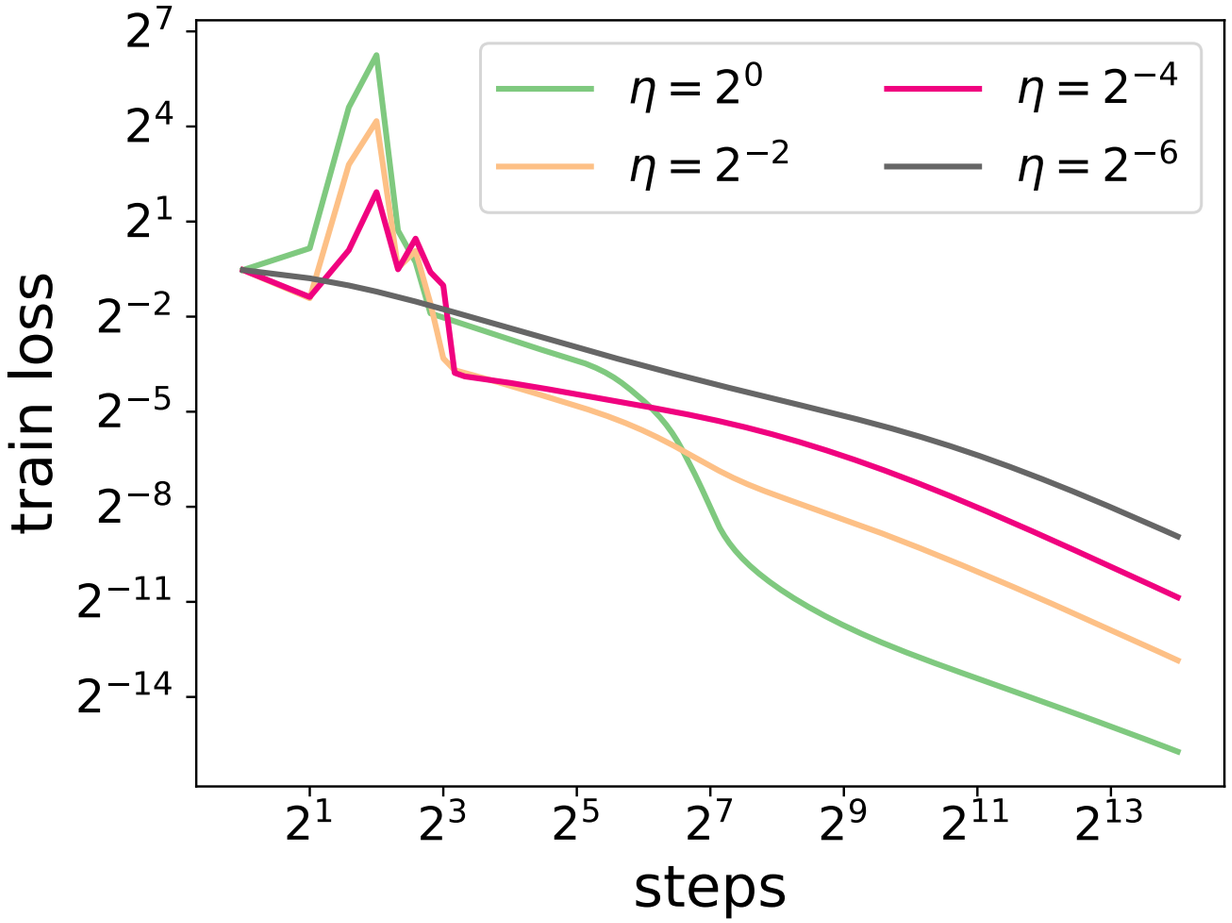


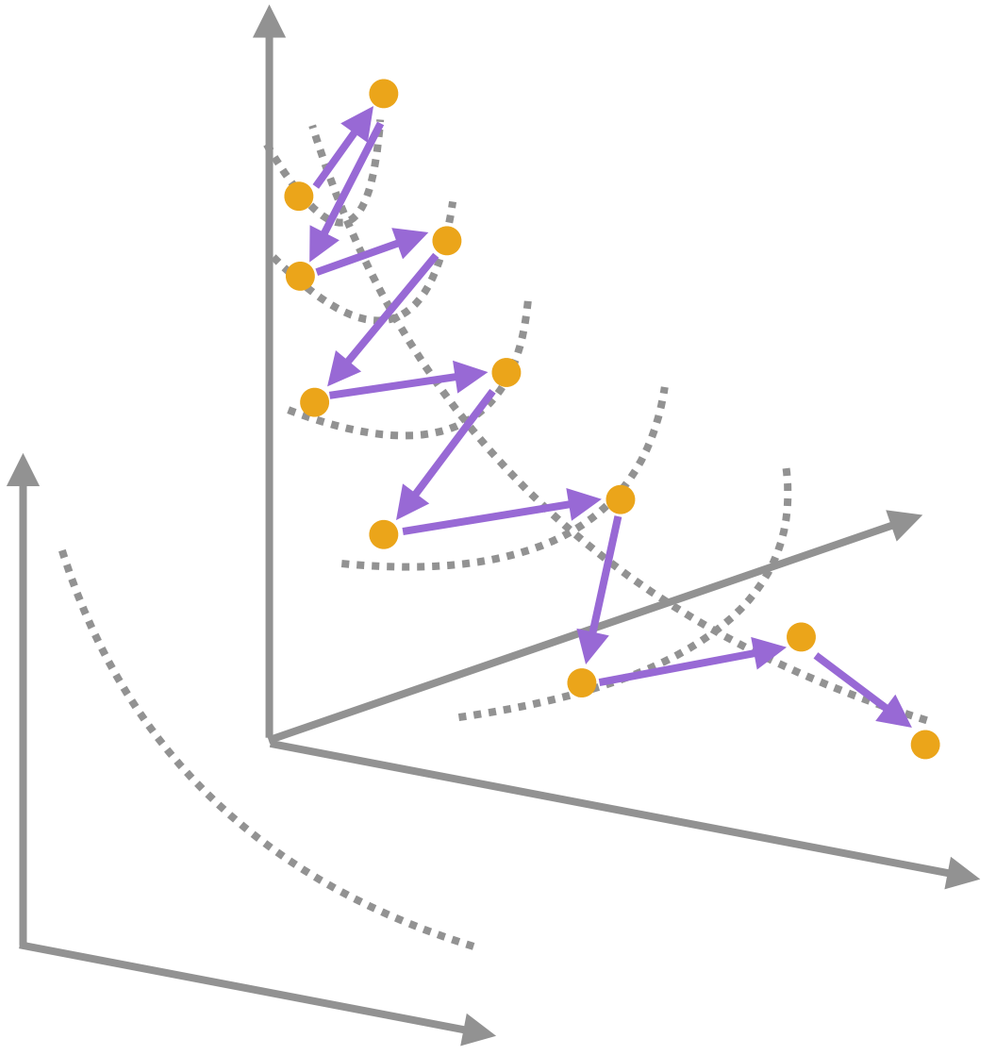

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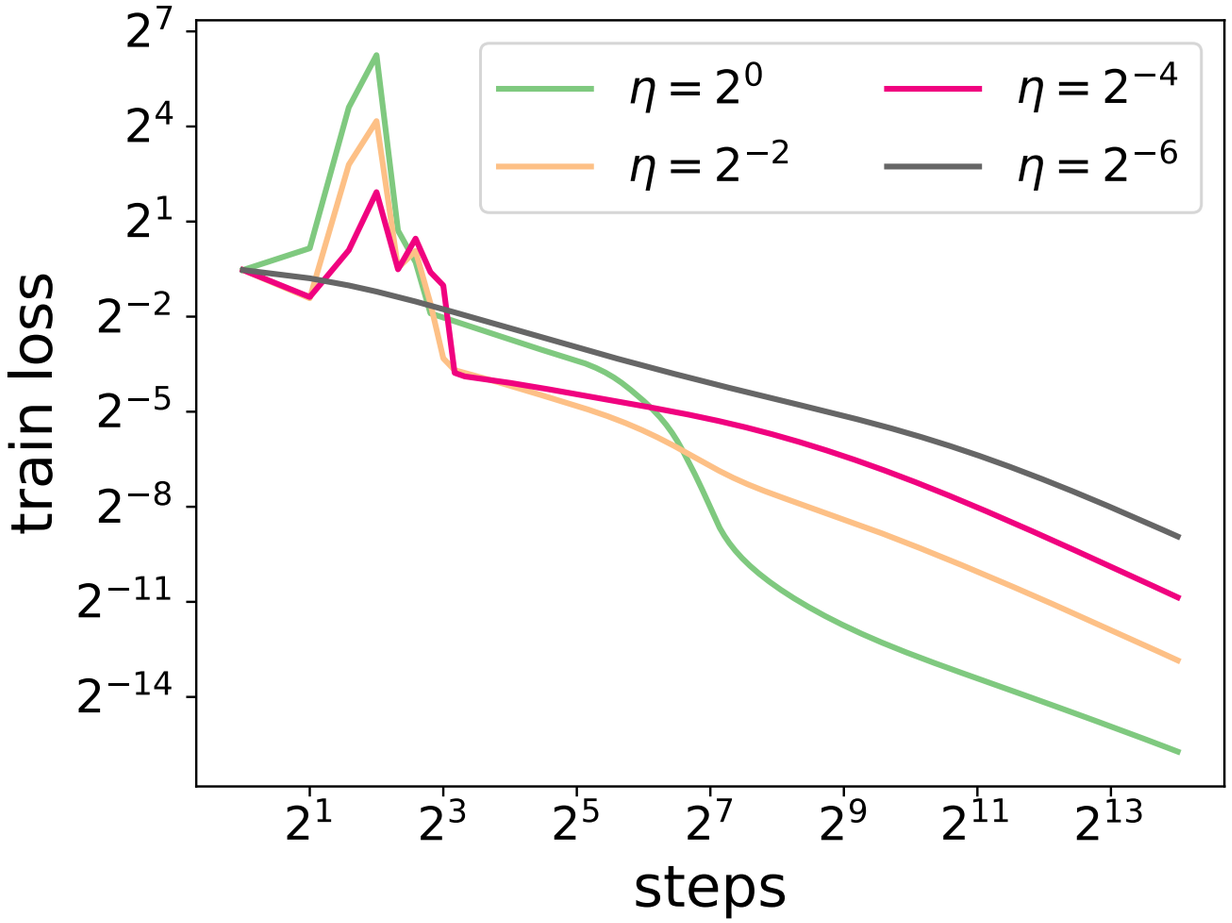


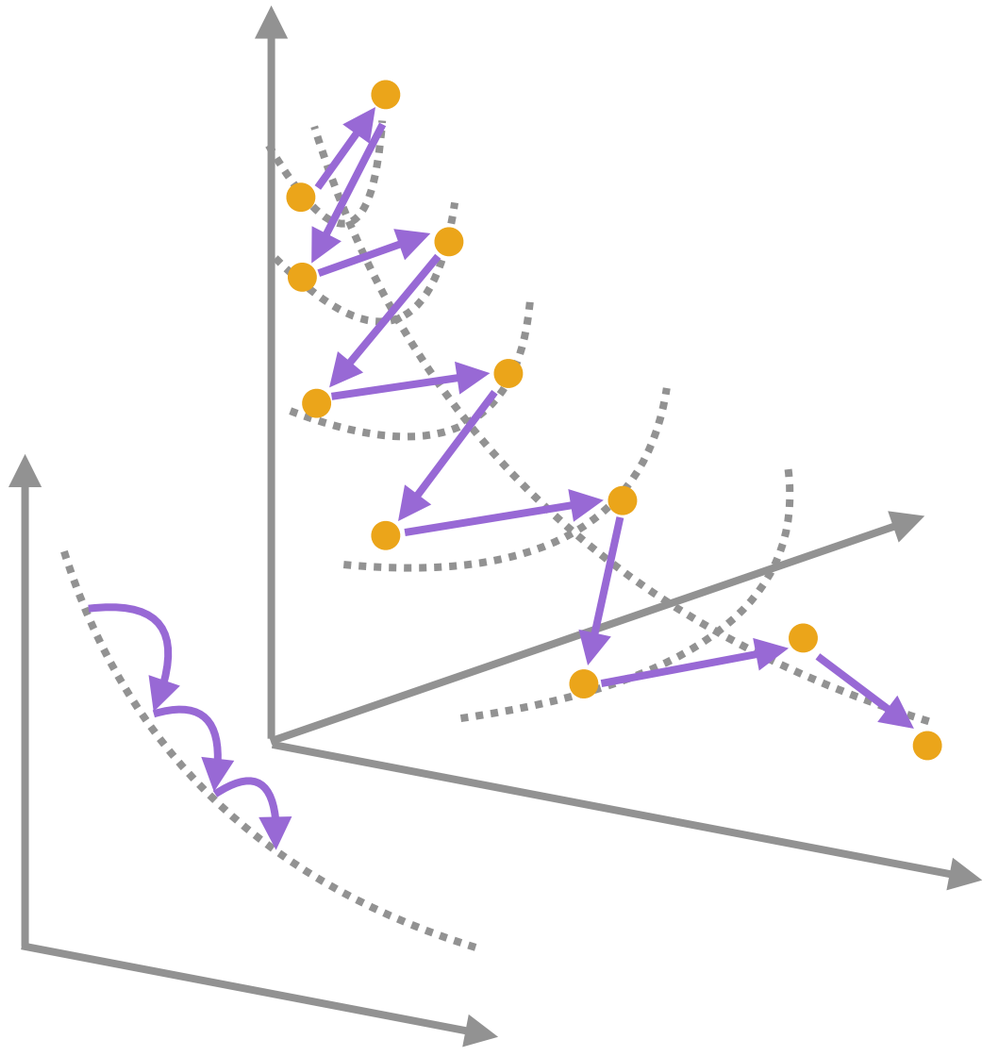

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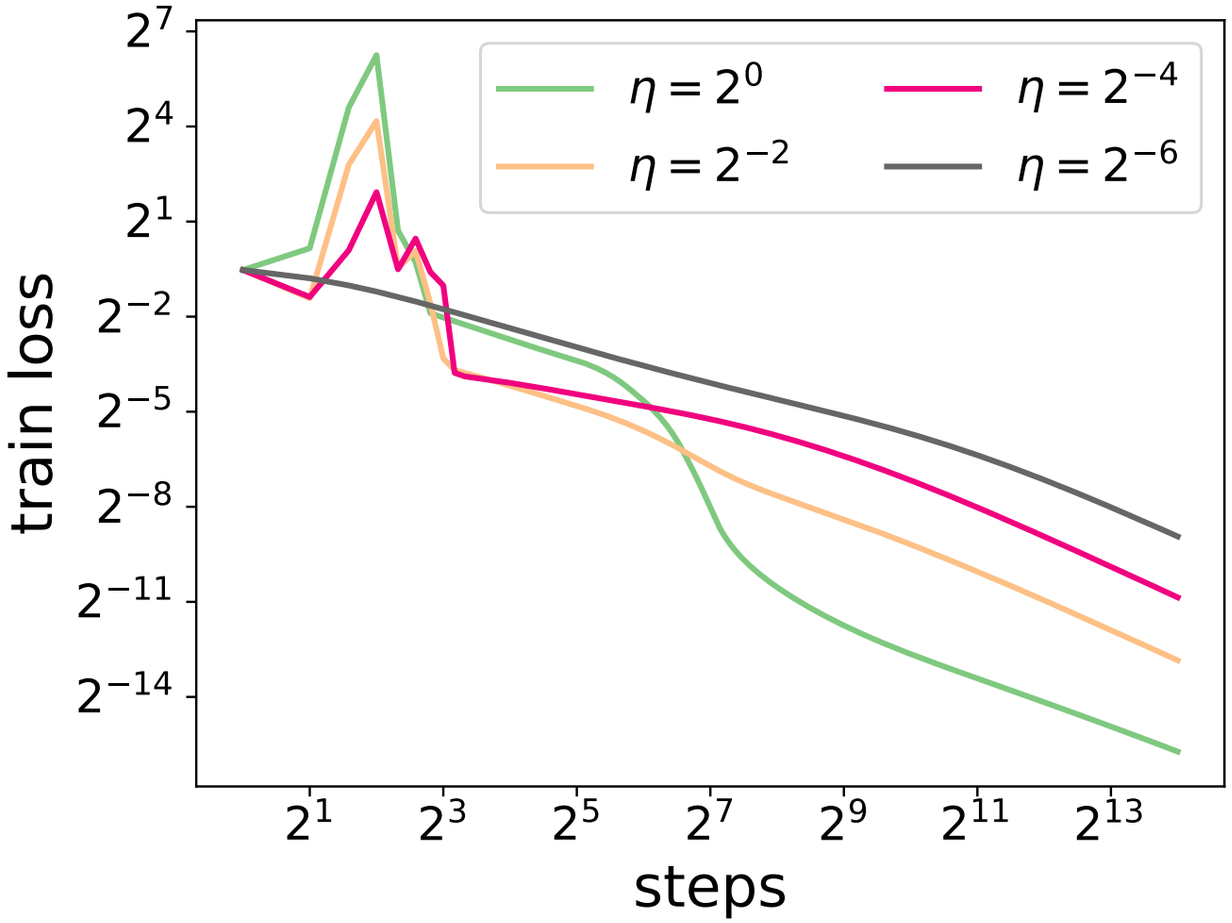
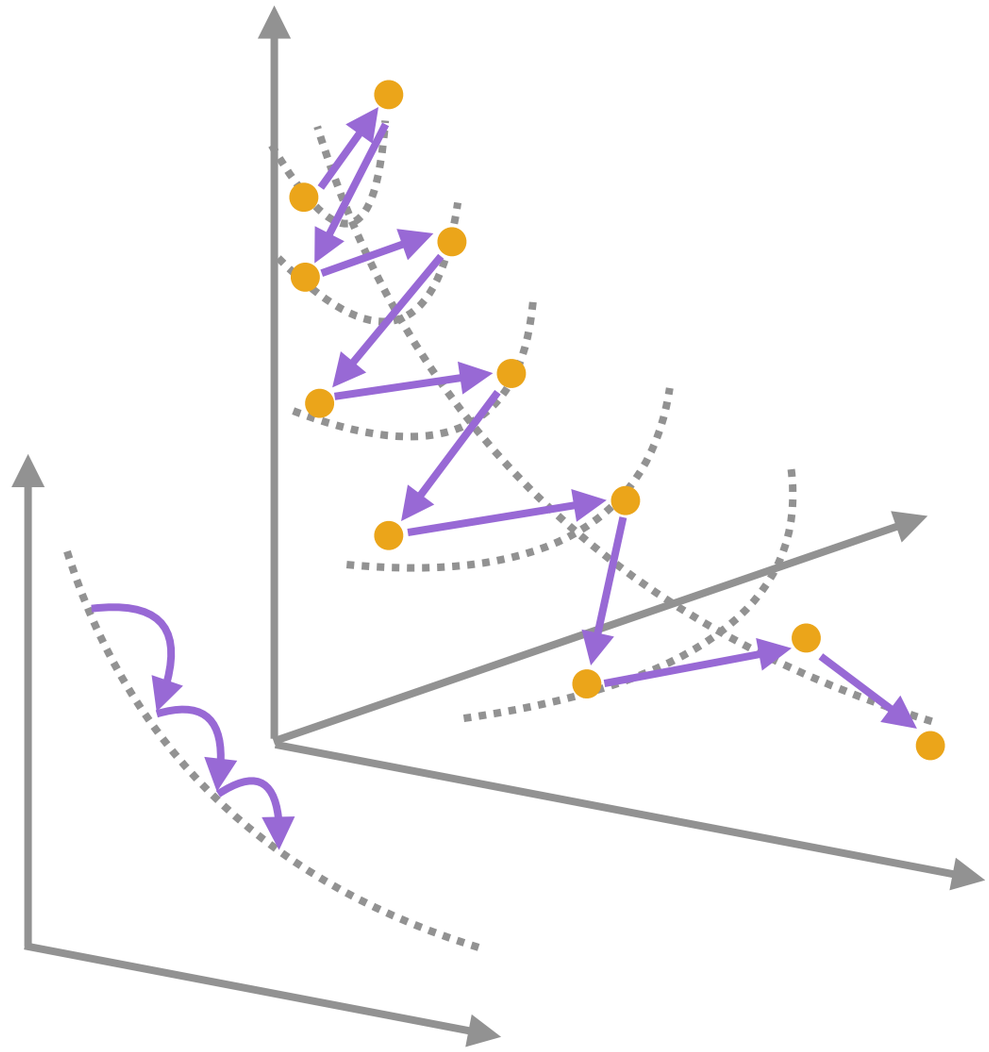
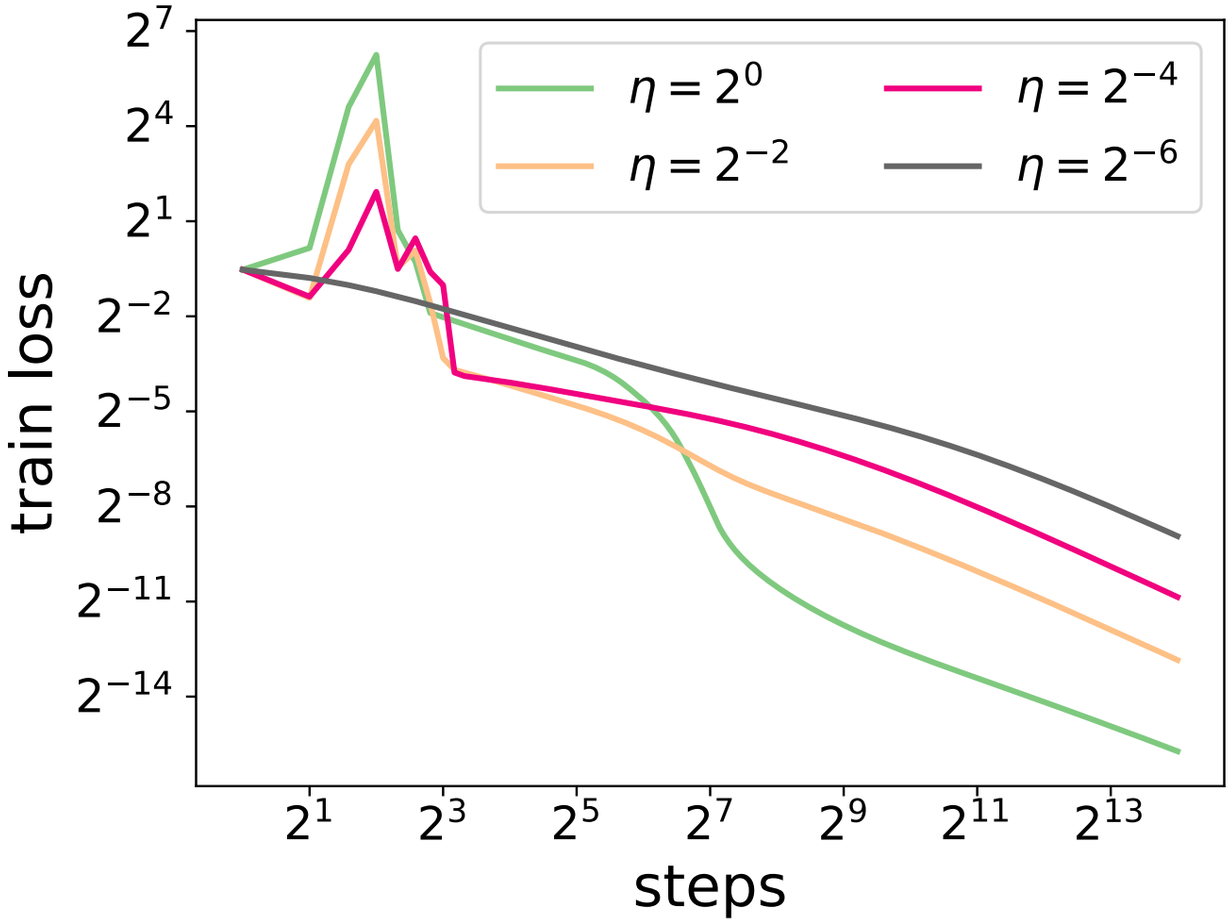


"open valley" as mental picture

Wu, Bartlett, Telgarsky, Yu. "Large stepsize gradient descent for logistic loss: non-monotonicity of the loss improves optimization efficiency." COLT 2024

# More details

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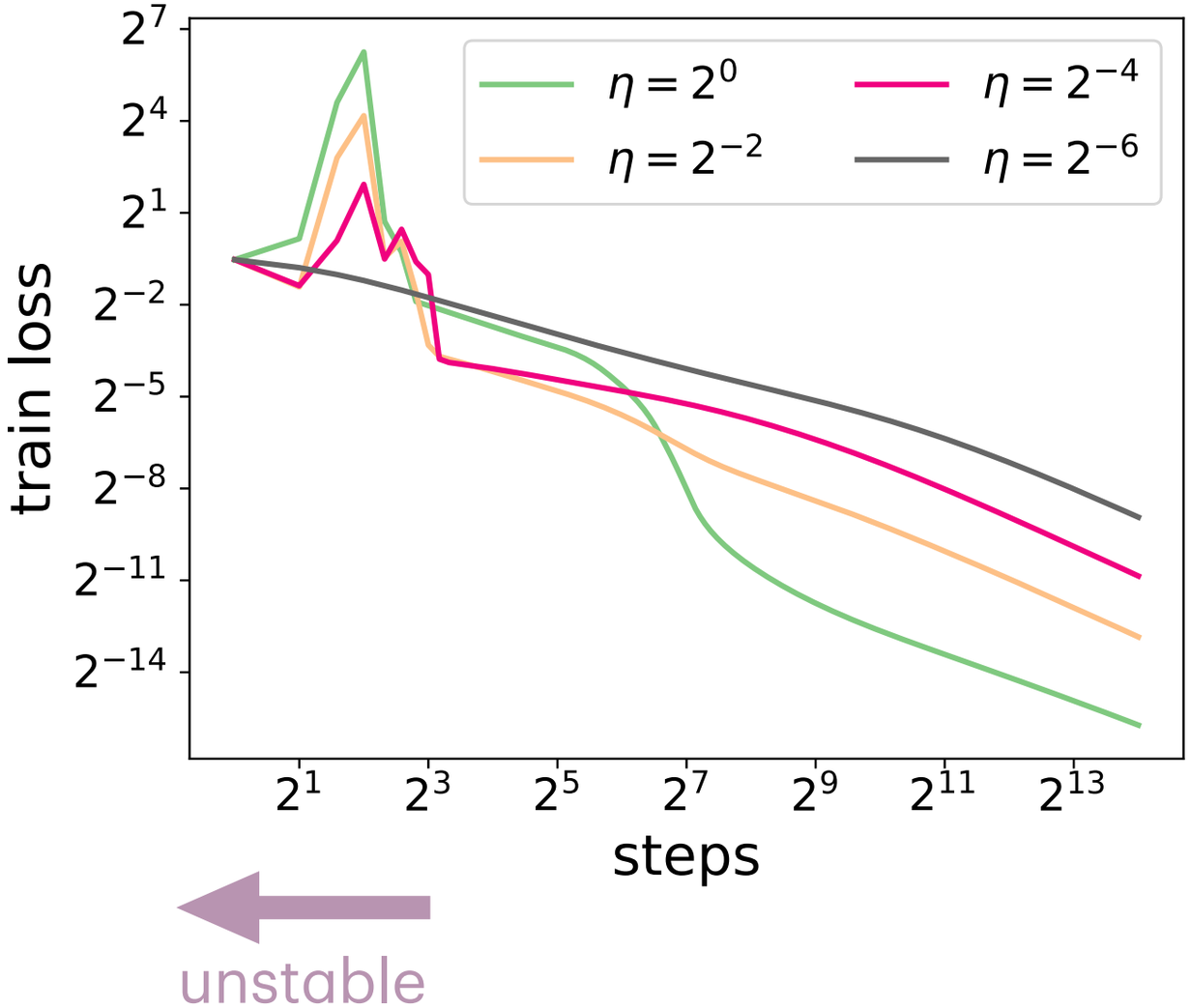


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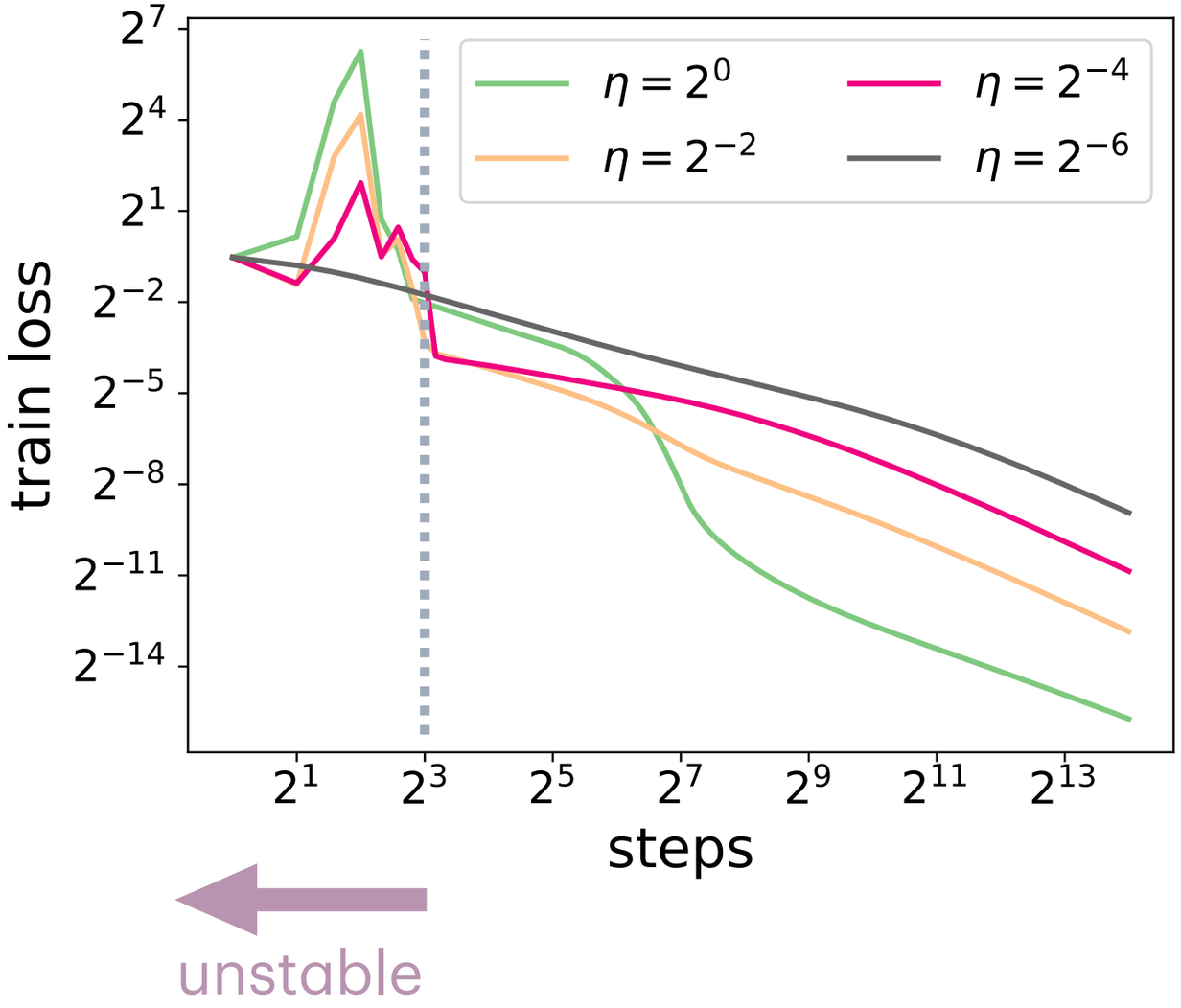


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**Wu**, Bartlett, Telgarsky, Yu. "Large stepsize gradient descent for logistic loss: non-monotonicity of the loss improves optimization efficiency." COLT 2024

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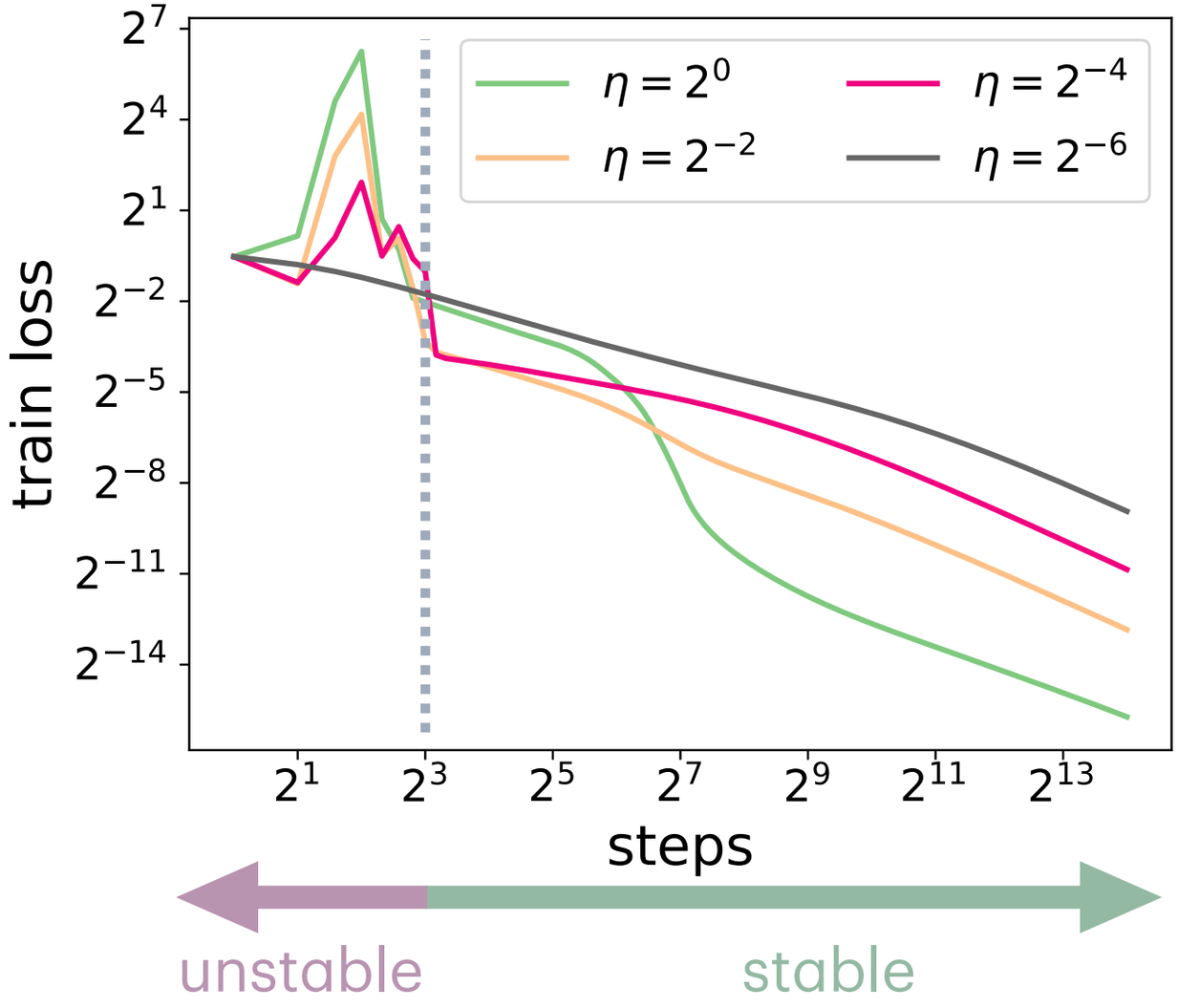


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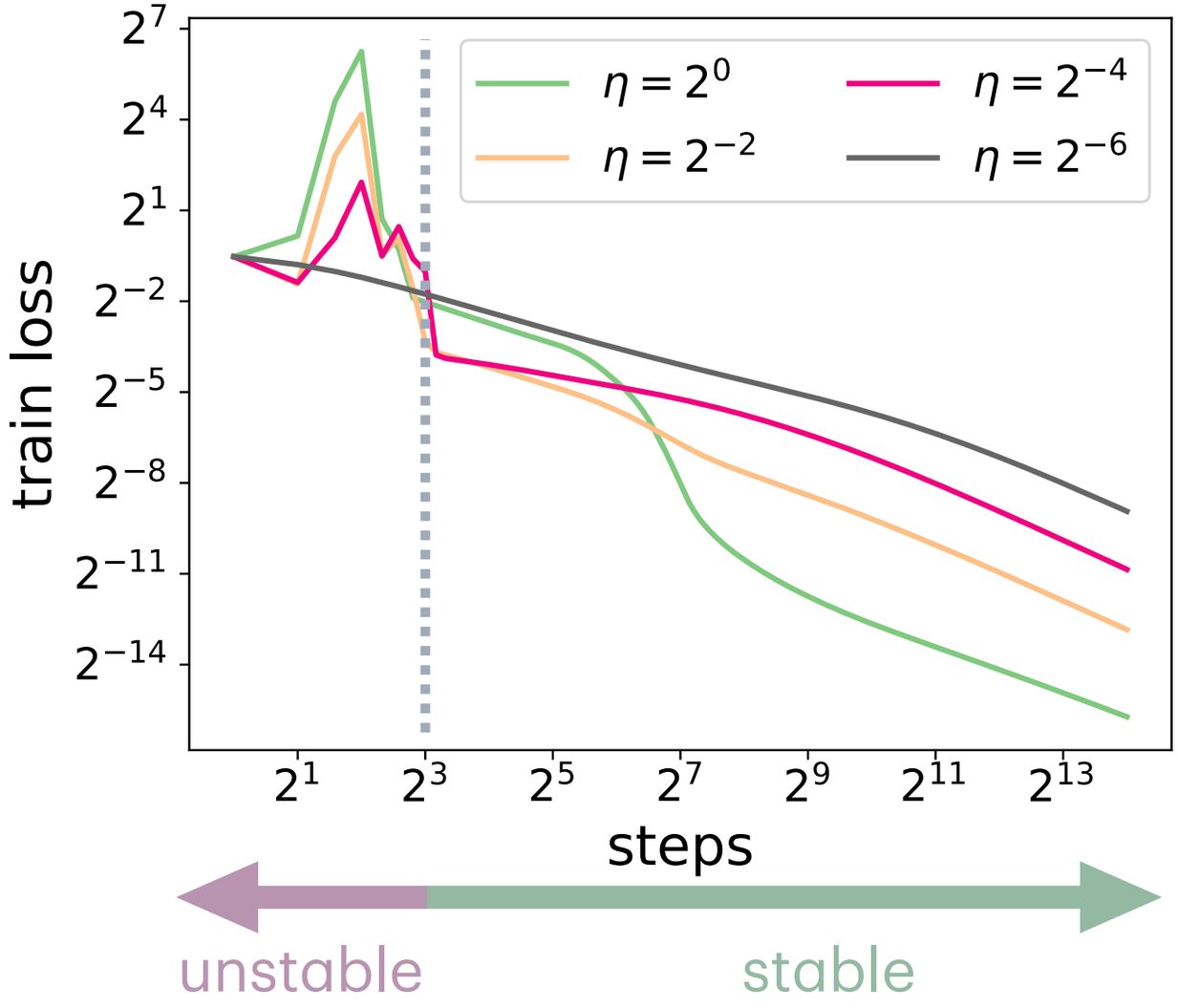
**Wu**, Bartlett, Telgarsky, Yu. "Large stepsize gradient descent for logistic loss: non-monotonicity of the loss improves optimization efficiency." COLT 2024

# More details

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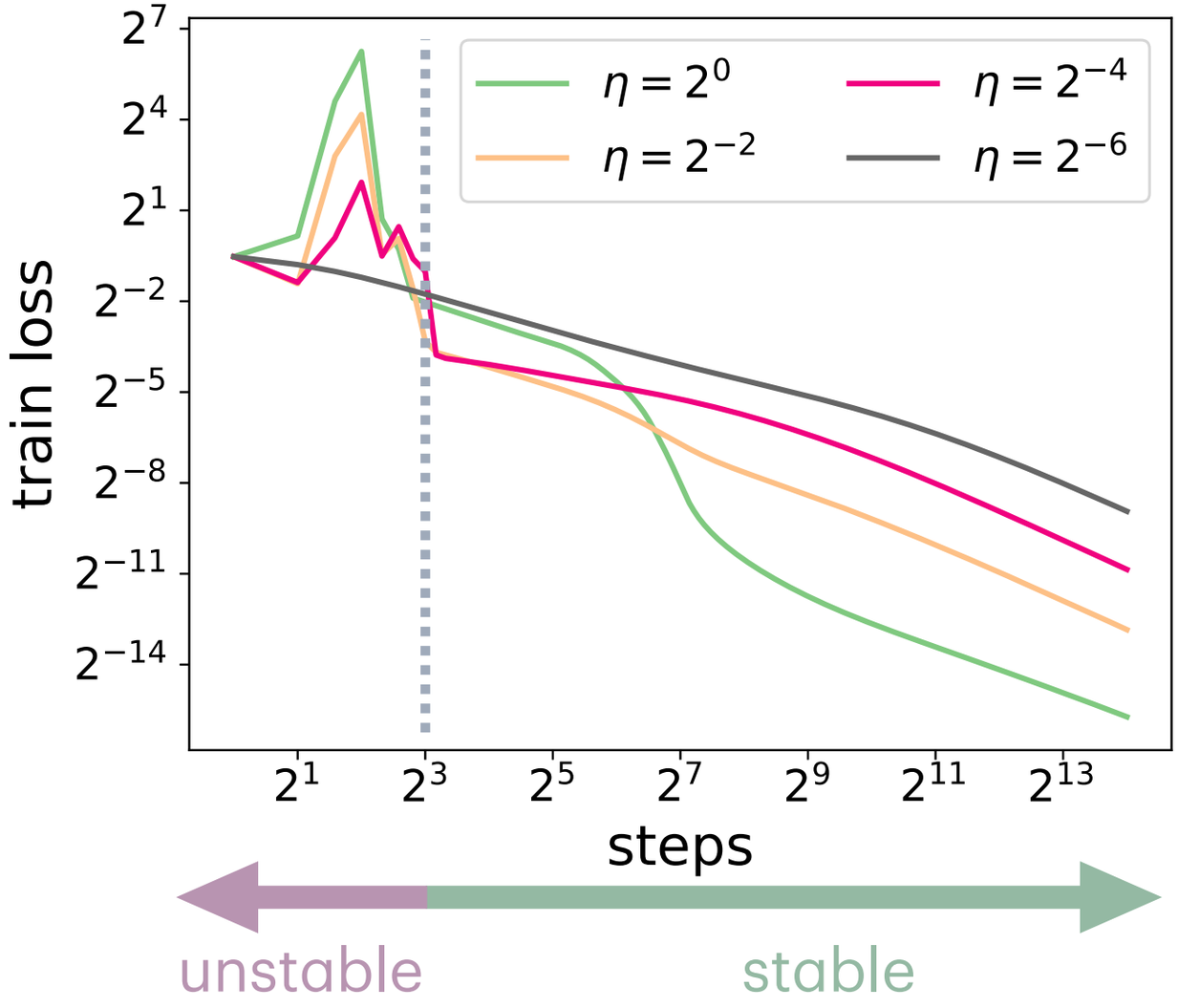


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"instability" is needed for acceleration

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# Regularized logistic regression

regularized empirical risk $\tilde{L}(\theta) = L(\theta) + \frac{\lambda}{2}||\theta||^2$

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match Nesterov via large stepsize

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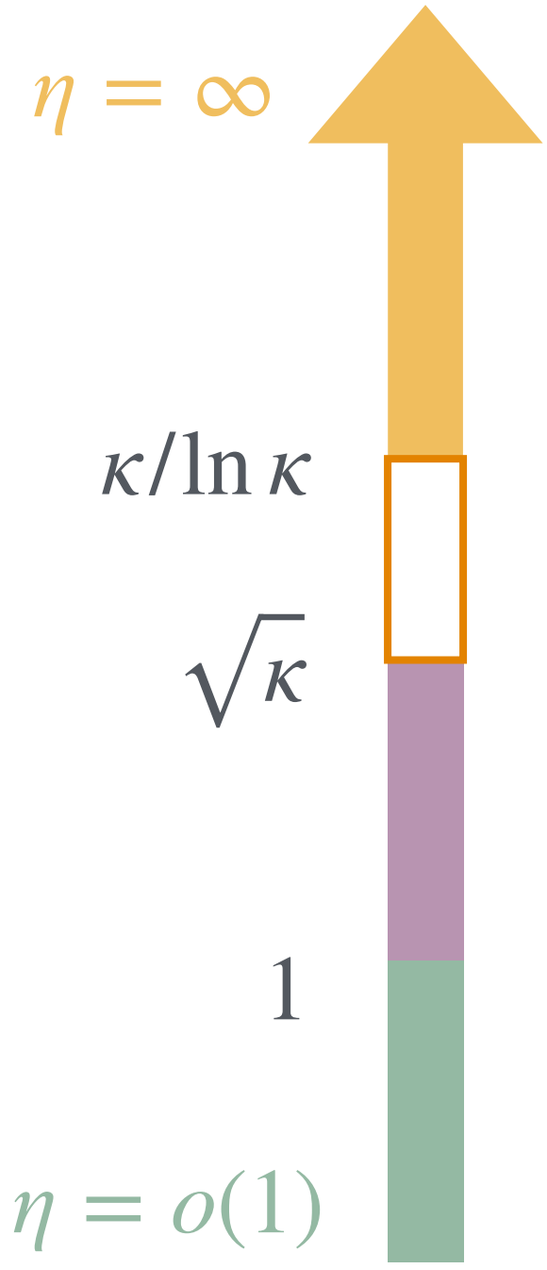


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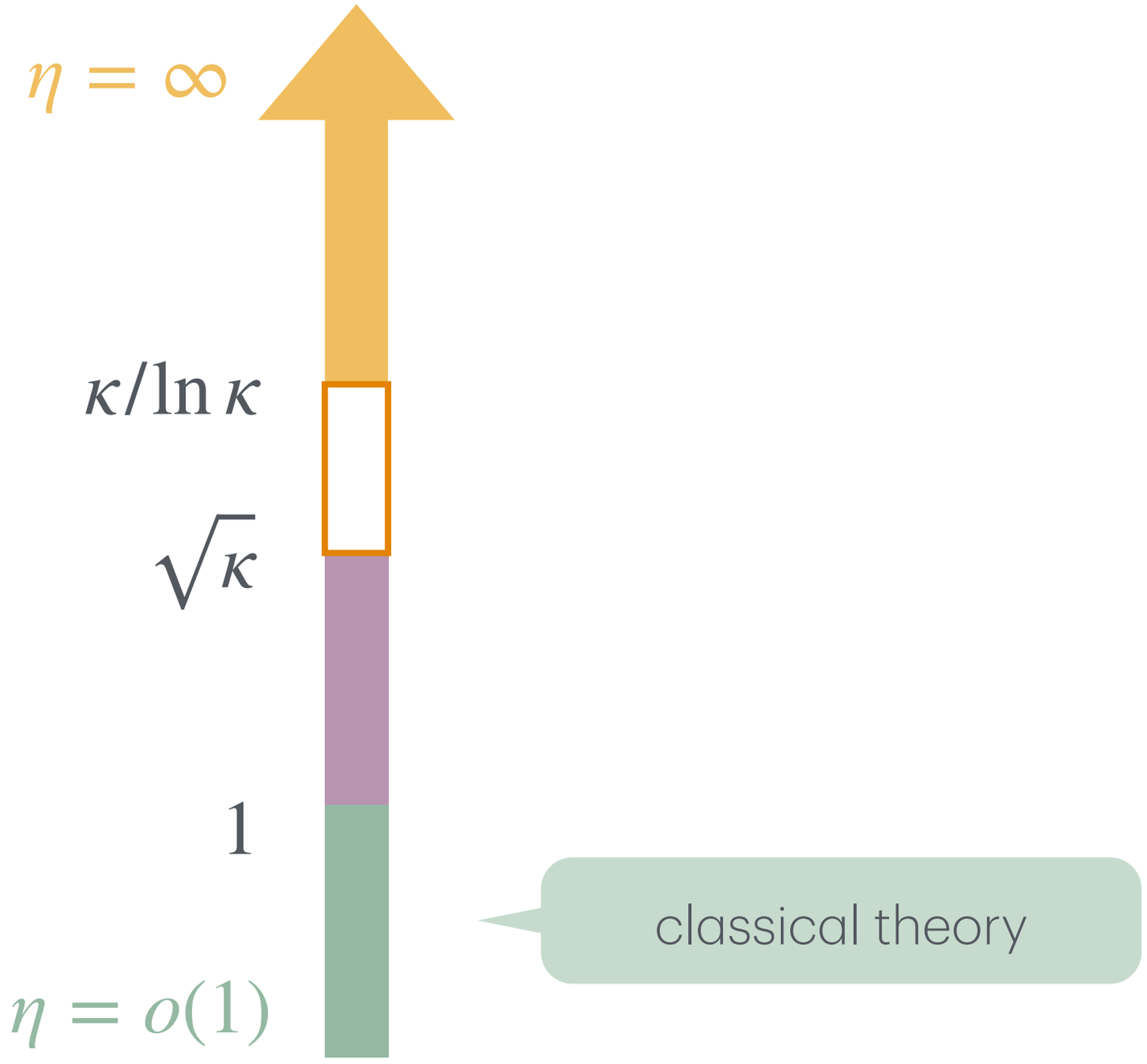


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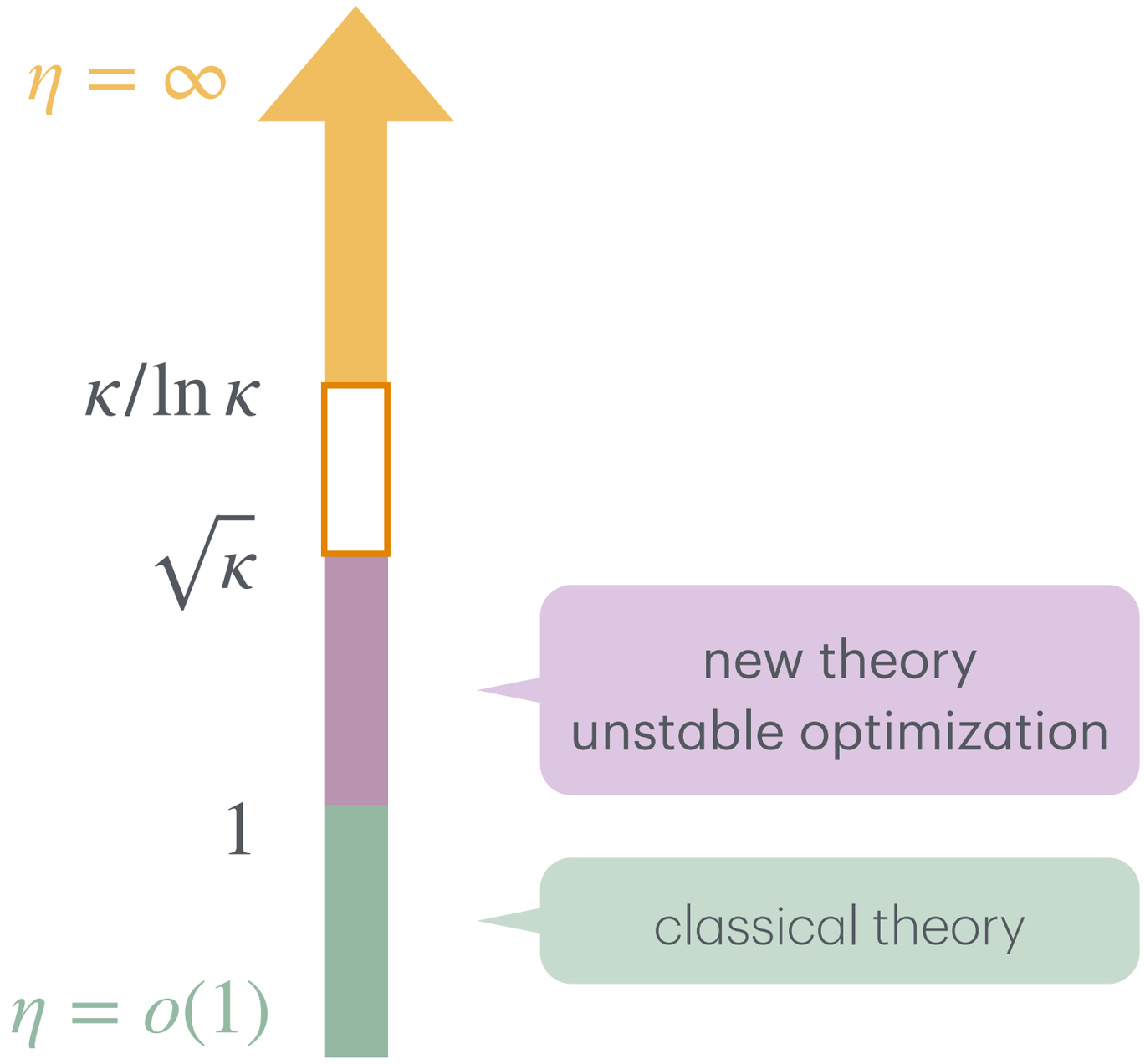


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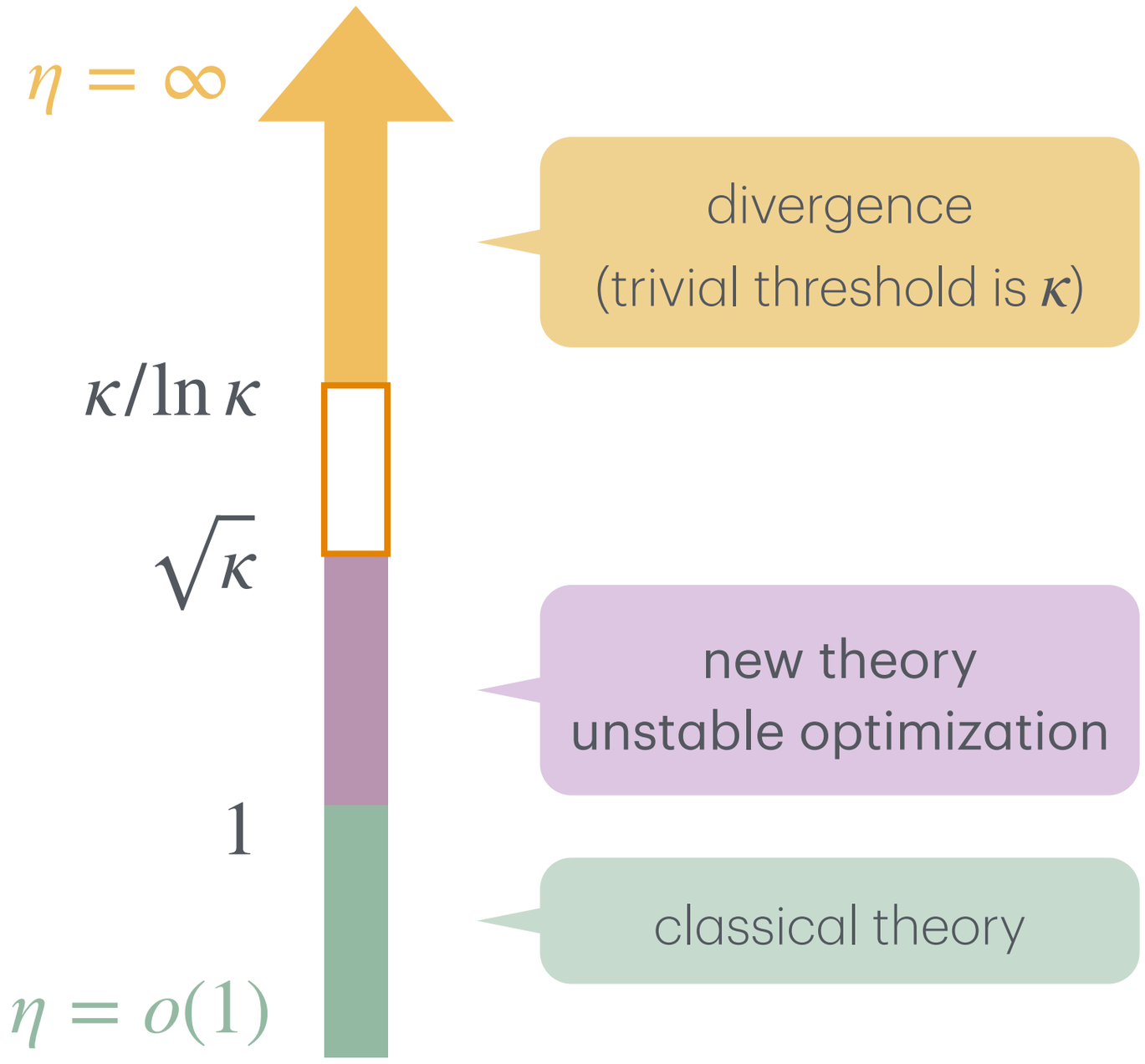


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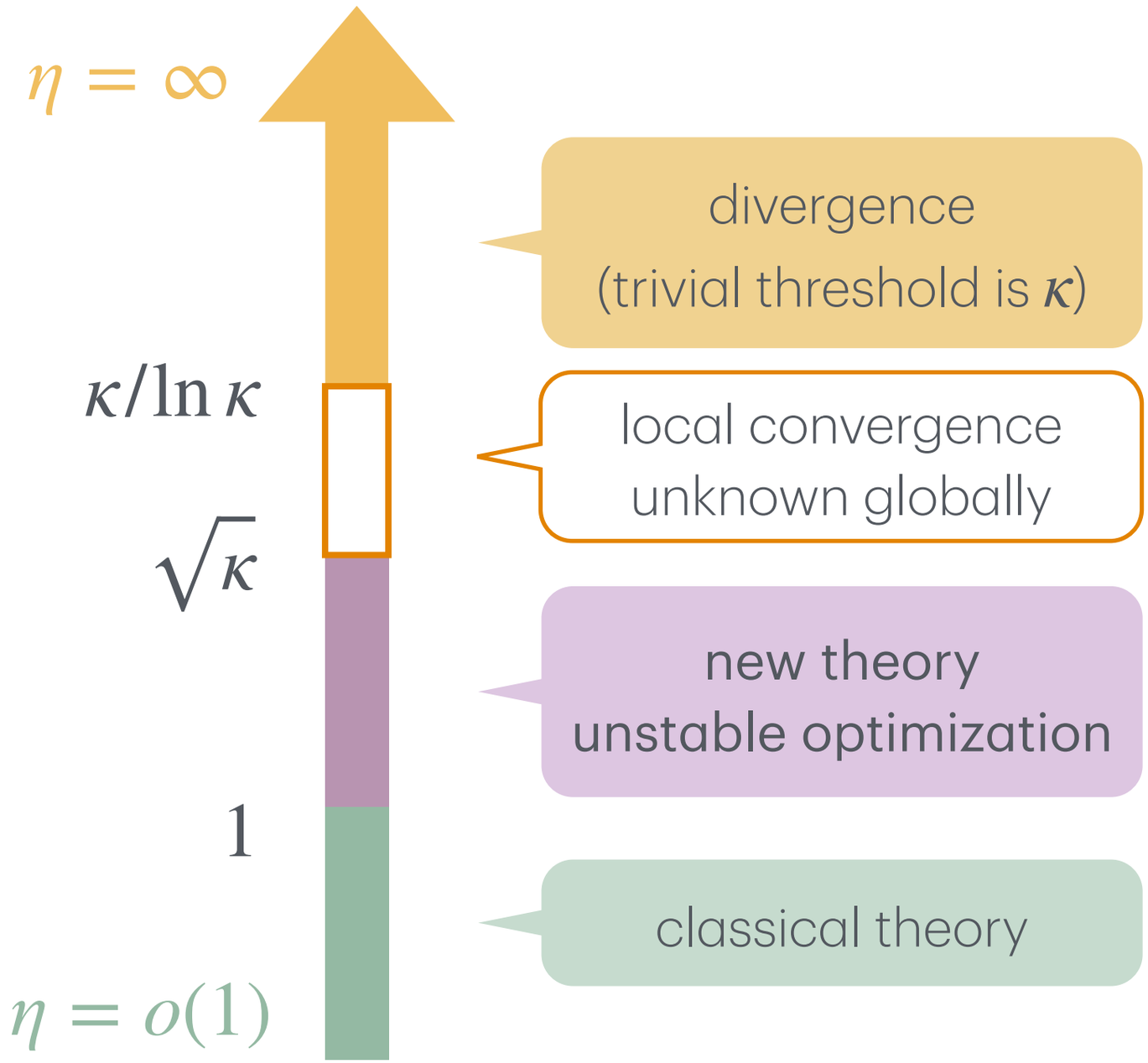


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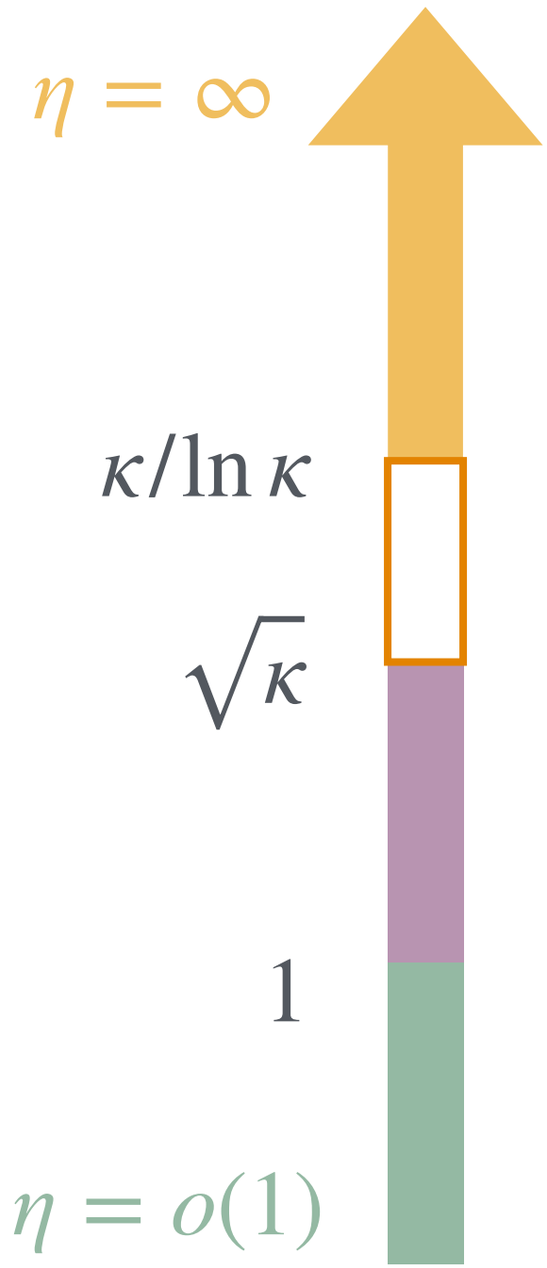


divergence
(trivial threshold is $\kappa$)

local convergence
unknown globally

new theory
unstable optimization

classical theory

three extensions
- general loss functions (WBTY'24)
- *two-layer networks (WBTY'24, CWMLB'24)
- large adaptive stepsizes (ZWLB'25)


**Wu**, Marion, Bartlett. "Large stepsizes accelerate gradient descent for regularized logistic regression." NeurIPS 2025

Cai, **Wu**, Mei, Lindsey, Bartlett. "Large stepsize GD for non-homogeneous two-layer networks: margin improvement and fast optimization." NeurIPS 2024

Zhang, **Wu**, Lin, Bartlett. "Minimax optimal convergence of gradient descent in logistic regression via large and adaptive stepsizes." ICML 2025

# Contribution 2: implicit regularization

gradient descent dominates ridge regression in linear regression

- “Risk comparisons in linear regression: implicit regularization dominates explicit regularization”
  **W**, Peter Bartlett, Sham Kakade, Jason Lee, Bin Yu
  COLT 2026

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Sandbox:
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early-stopped GD is

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$n$ iid samples $(x_1, y_1), \ldots, (x_n, y_n)$

$$X = \begin{bmatrix} x_1^\top \\ \vdots \\ x_n^\top \end{bmatrix} \quad Y = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$$

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fix $0 < \eta \leq 1/\|\nabla^2 L\|$; otherwise, rescale time

# GD dominates ridge

**Theorem.** For **every** $(\Sigma, \theta^*)$, $n \geq 1$, $\lambda \geq 0$, there exists $t \geq 0$ such that, w.p. $\geq 0.99$

$$\mathsf{risk}\left(\hat{\theta}_t^{\mathsf{gd}}\right) \lesssim \mathsf{risk}\left(\hat{\theta}_\lambda^{\mathsf{ridge}}\right)$$

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**Prior work.** Assume $\mathbb{E}\theta^* = 0$, $\mathbb{E}\theta^{*\otimes 2} \propto I$. For $t \propto 1/\lambda$, we have

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e.g., $\eta t = 1/\lambda$

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There exists $(\Sigma, \theta^*)$ and $t_n$ such that for **any** $\lambda$, w.p. $\geq 0.99$

natural problems
e.g., power-law

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**(TB'23)** There is an analytic function $f$ such that, w.p. $\geq 0.99$

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more effective when

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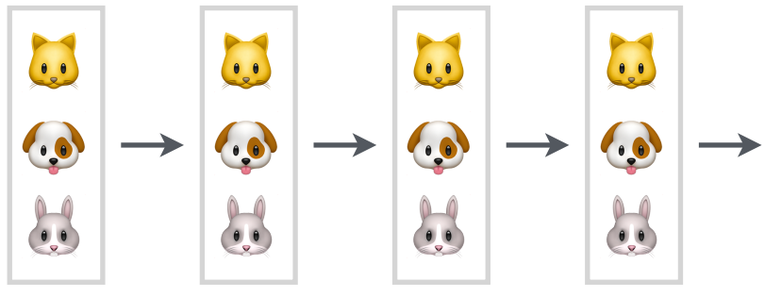

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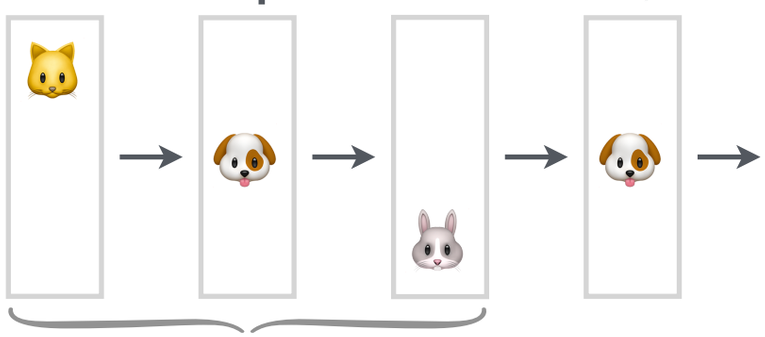


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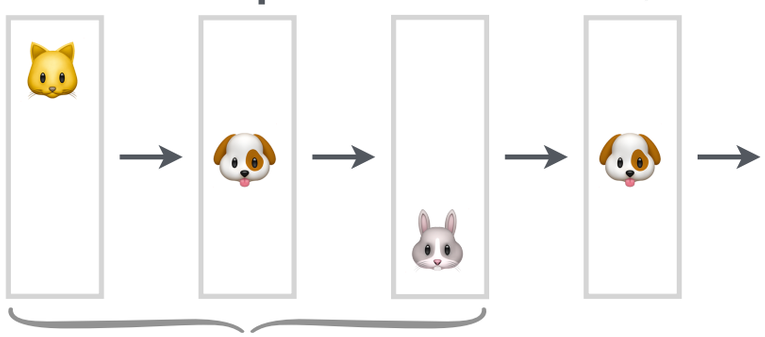


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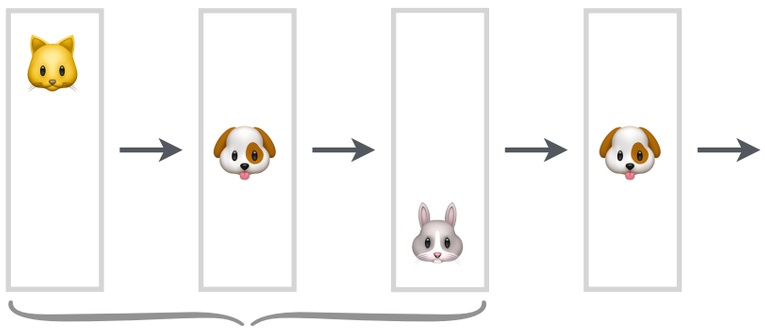

epoch 1

the SGD variant
used in deep learning

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# Contribution 3: from theory to practice

principled parallelization method for training language models

- "*Seesaw*: accelerating training by balancing learning rate and batch size scheduling"
  Alexandru Meterez*, Depen Morwani*, 𝕎, Costin-Andrei Oncescu, Cengiz Pehlevan, Sham Kakade
  ICLR 2026

# Language Model (LM) training

**Practice.** LM training is “online”: #data $\propto$ #flops

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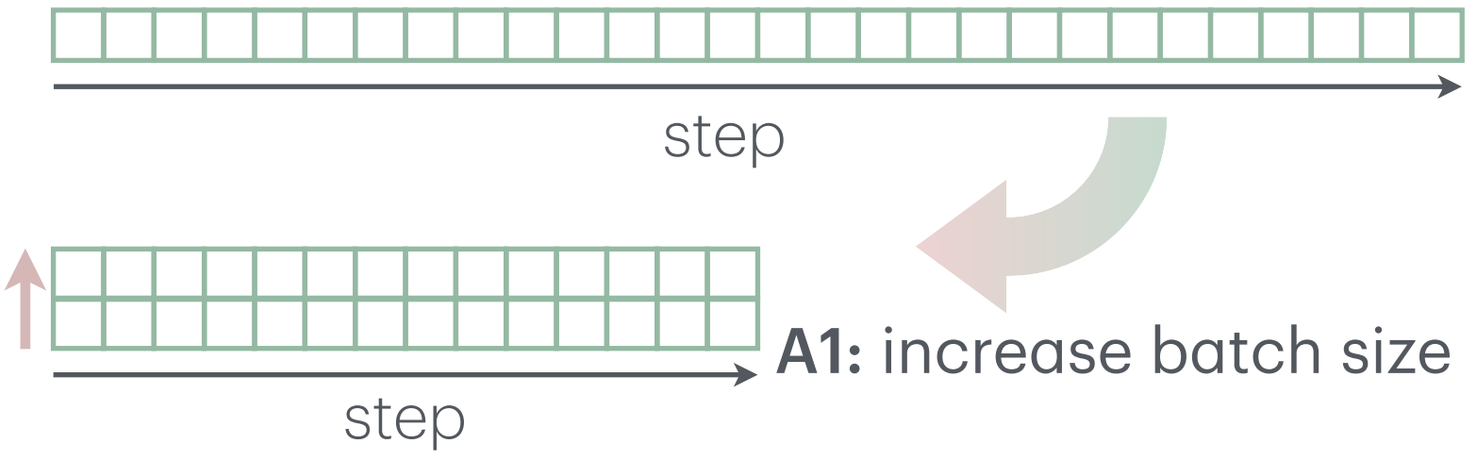


**A1:** increase batch size

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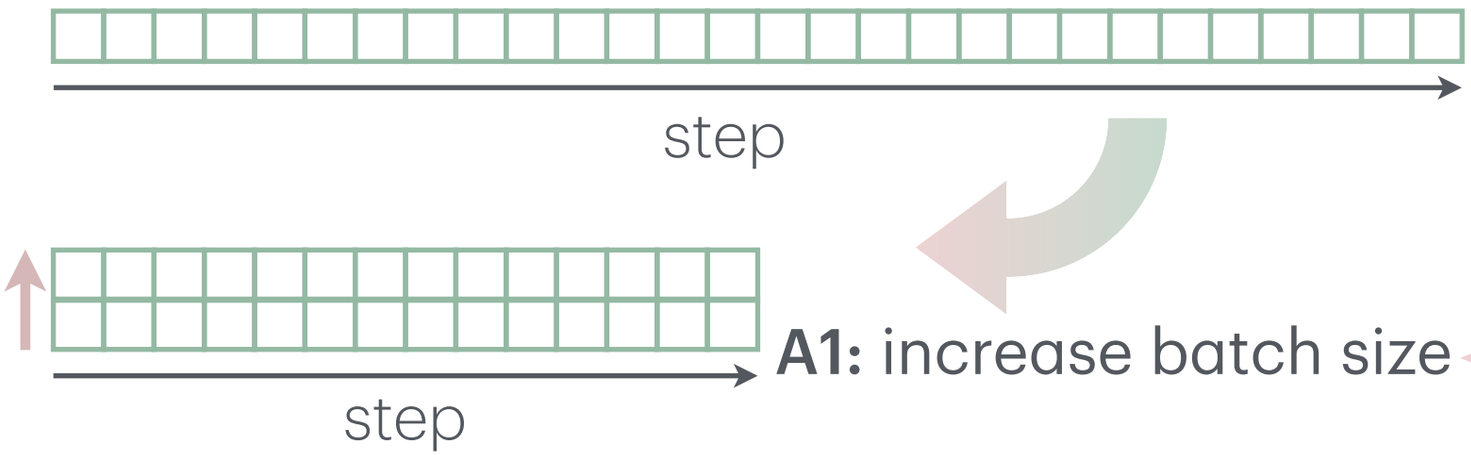


**A1:** increase batch size

cannot exceed
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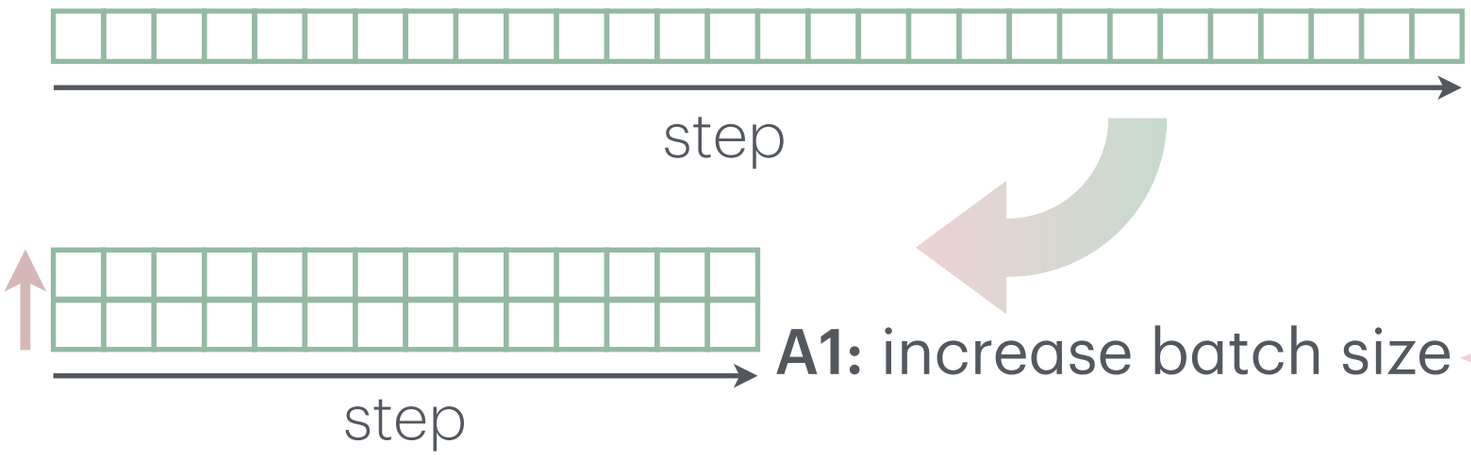


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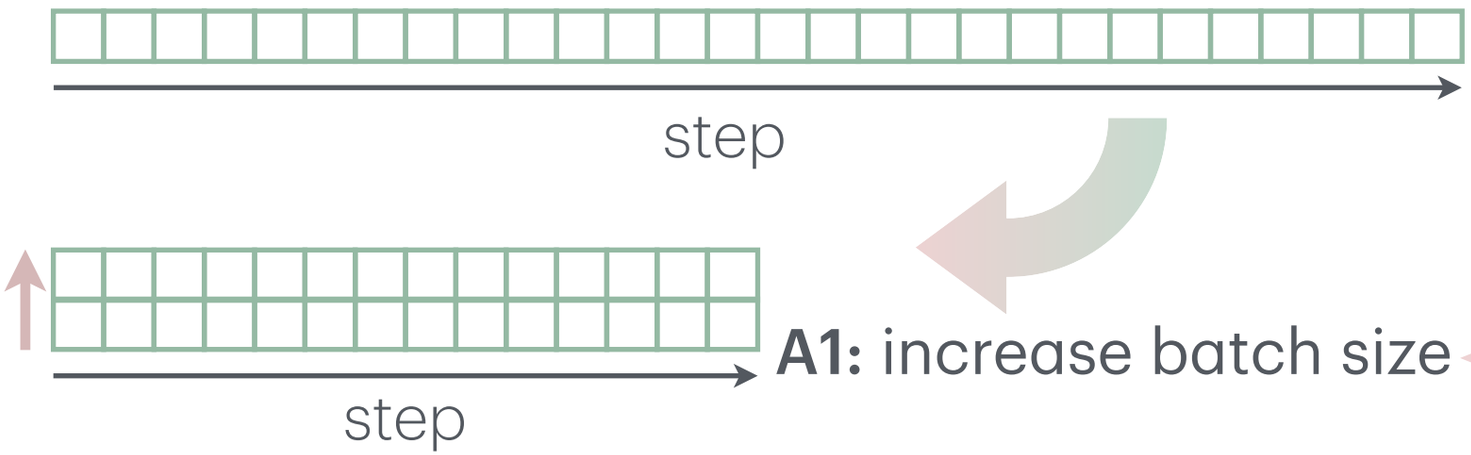


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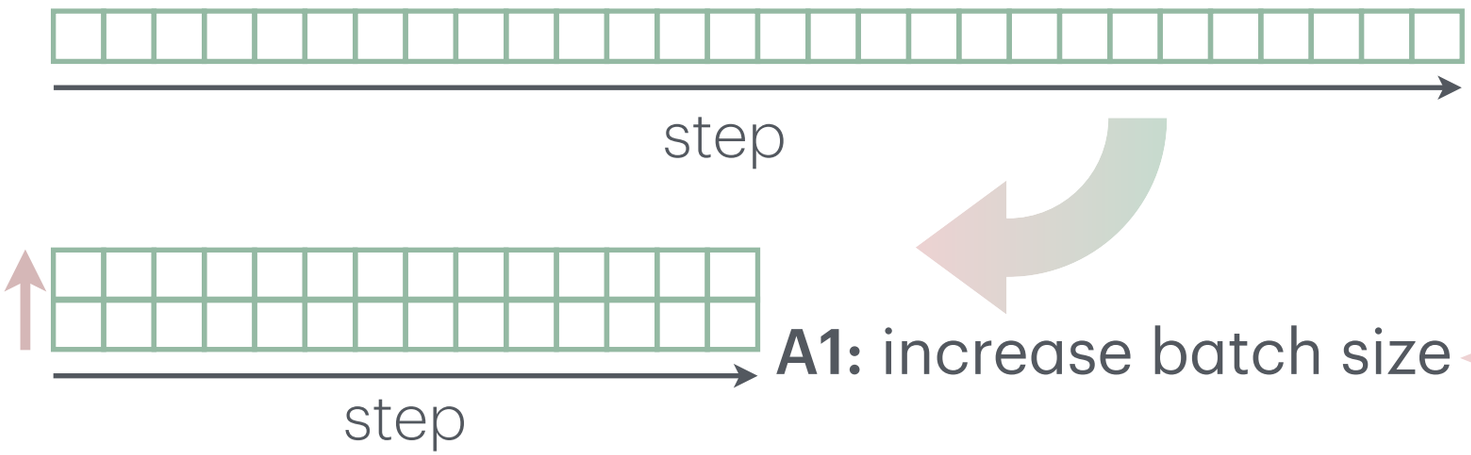


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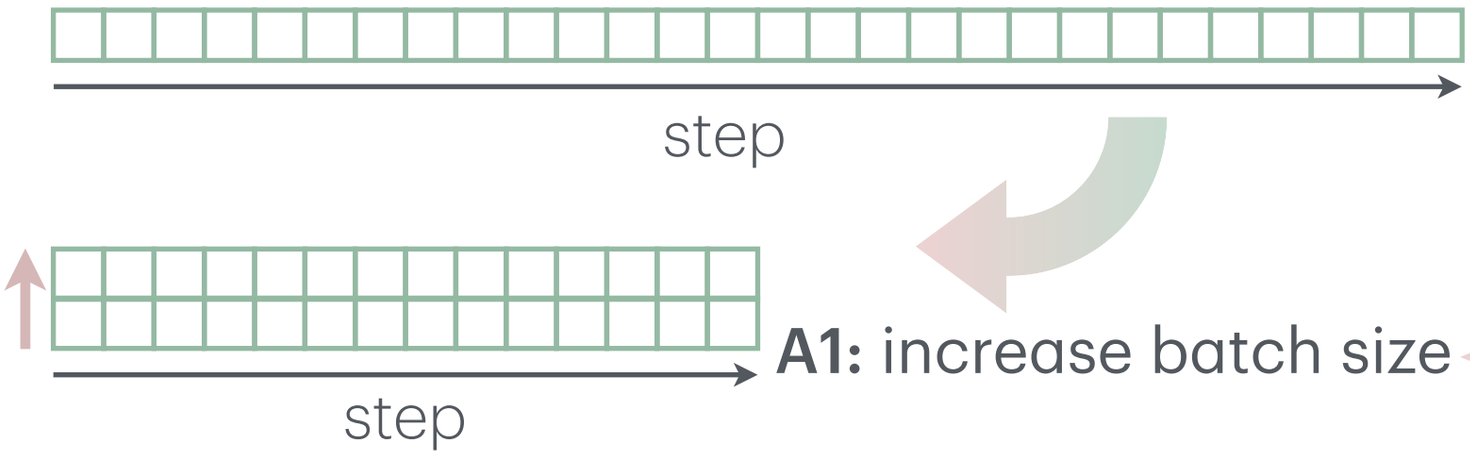


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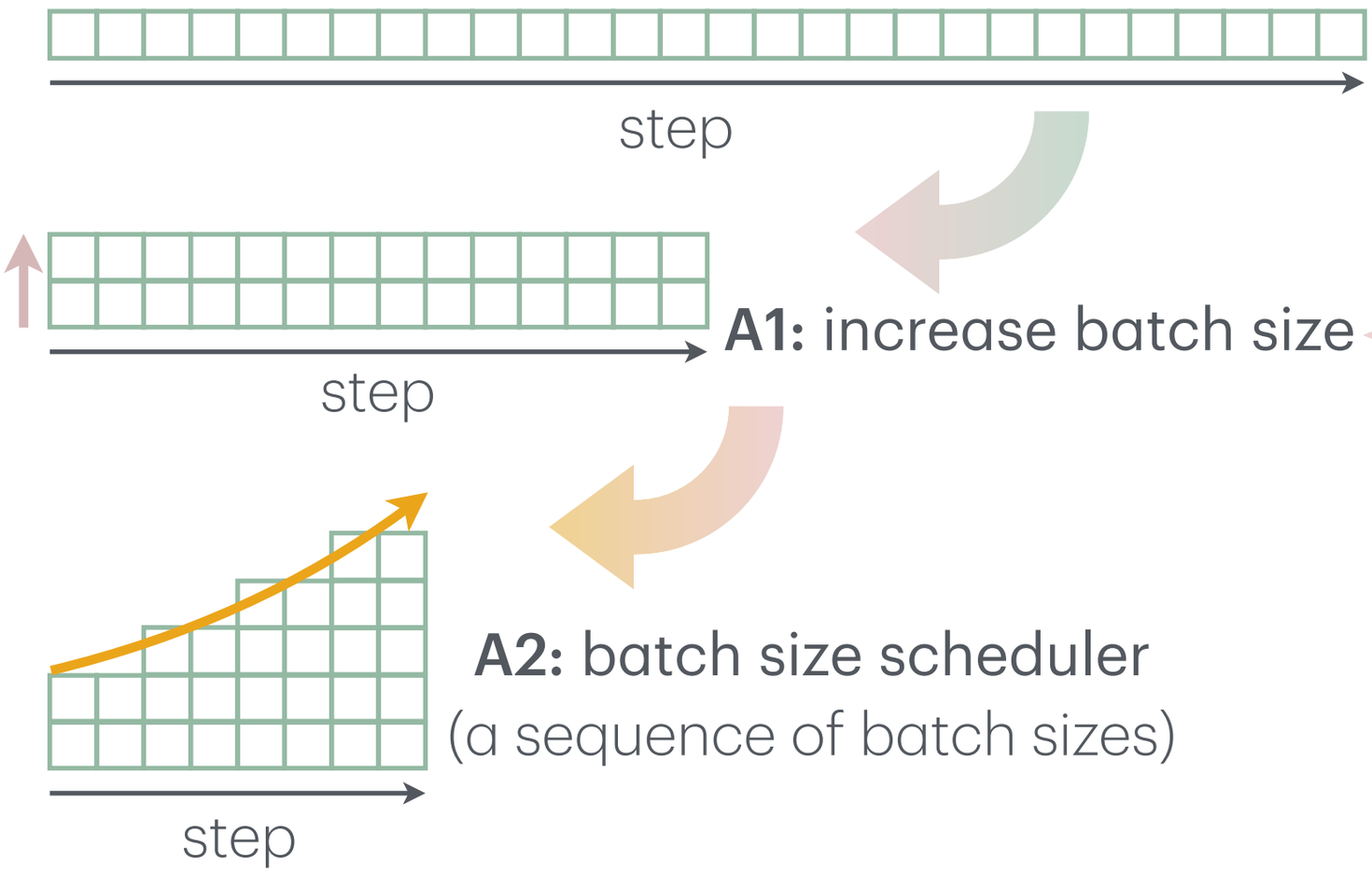


**A1:** increase batch size

**A2:** batch size scheduler
(a sequence of batch sizes)

cannot exceed
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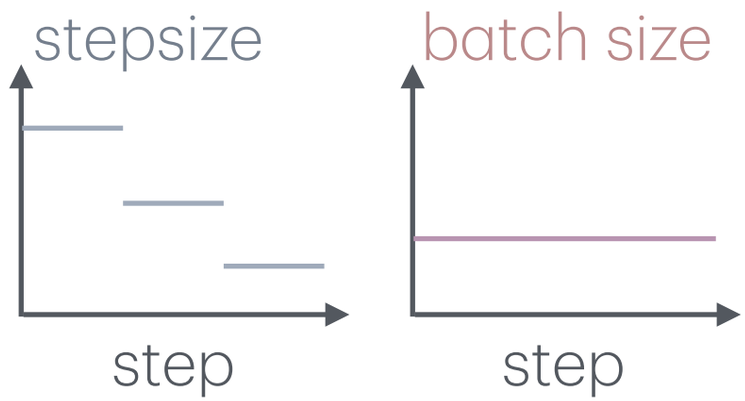


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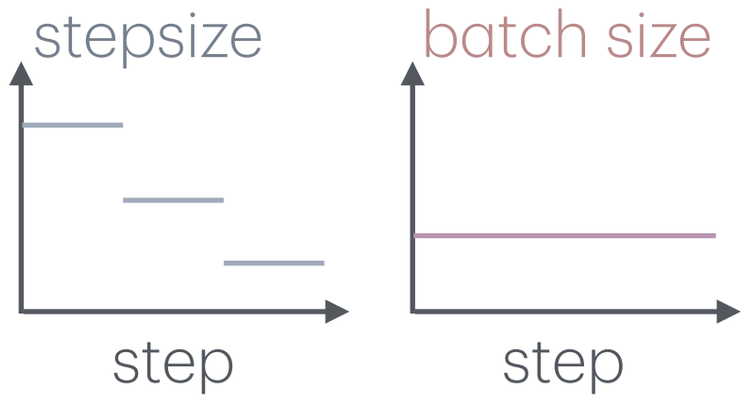
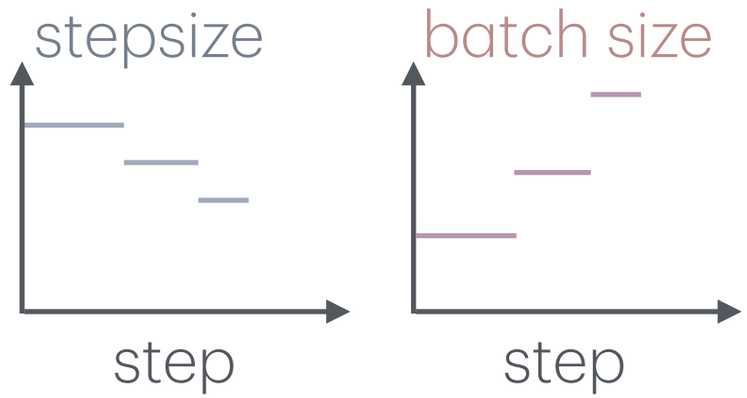


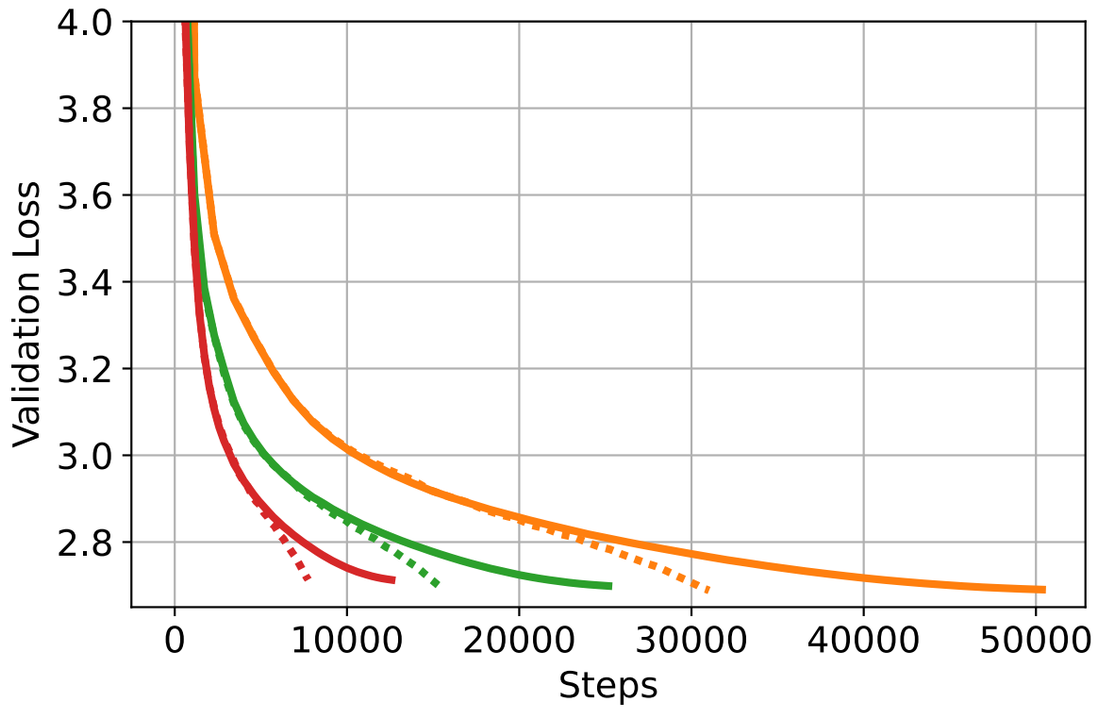

***Seesaw:*** joint scheduler

$\eta \to \eta/\sqrt{1.1}, B \to 1.1B$




Meterez, Morwani, **Wu**, Oncescu, Pehlevan, Kakade. "*Seesaw*: accelerating training by balancing learning rate and batch size scheduling." ICLR 2026

# Same error (±0.17%), 36% fewer steps




transformer (600M), Adam, C4

initial batch size: $2^8$, $2^9$, $2^{10}$ (= CBS)
solid curve: **default** (fixed batch size, cosine stepsize scheduler)
dotted curve: *Seesaw* (ours)

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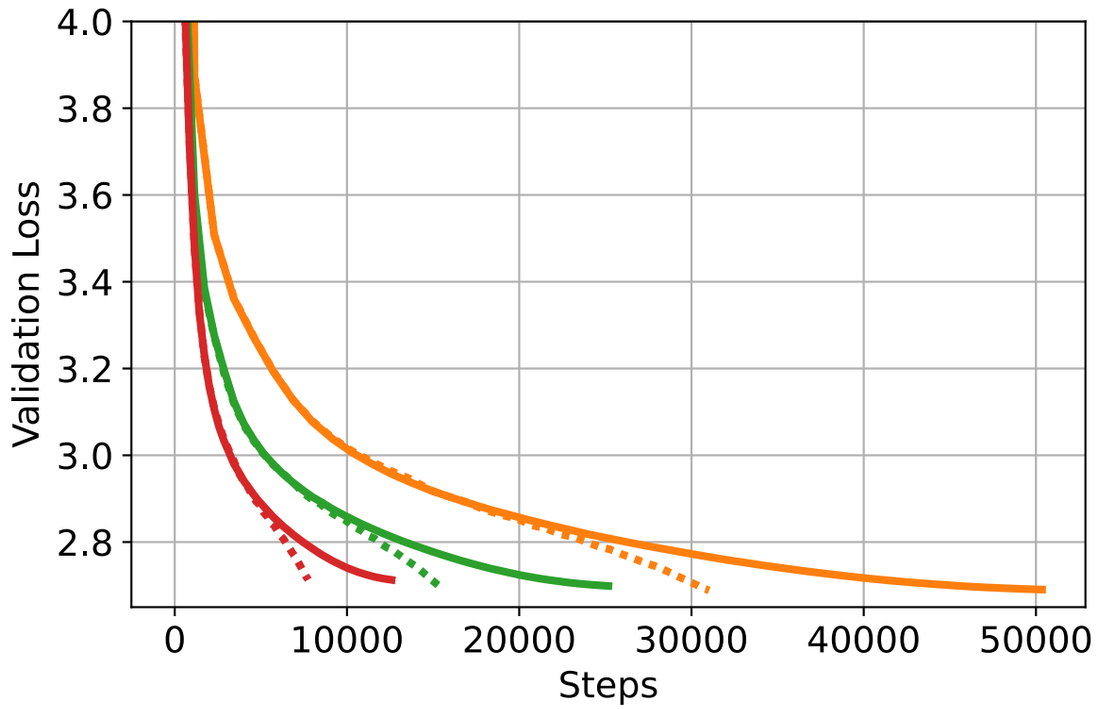


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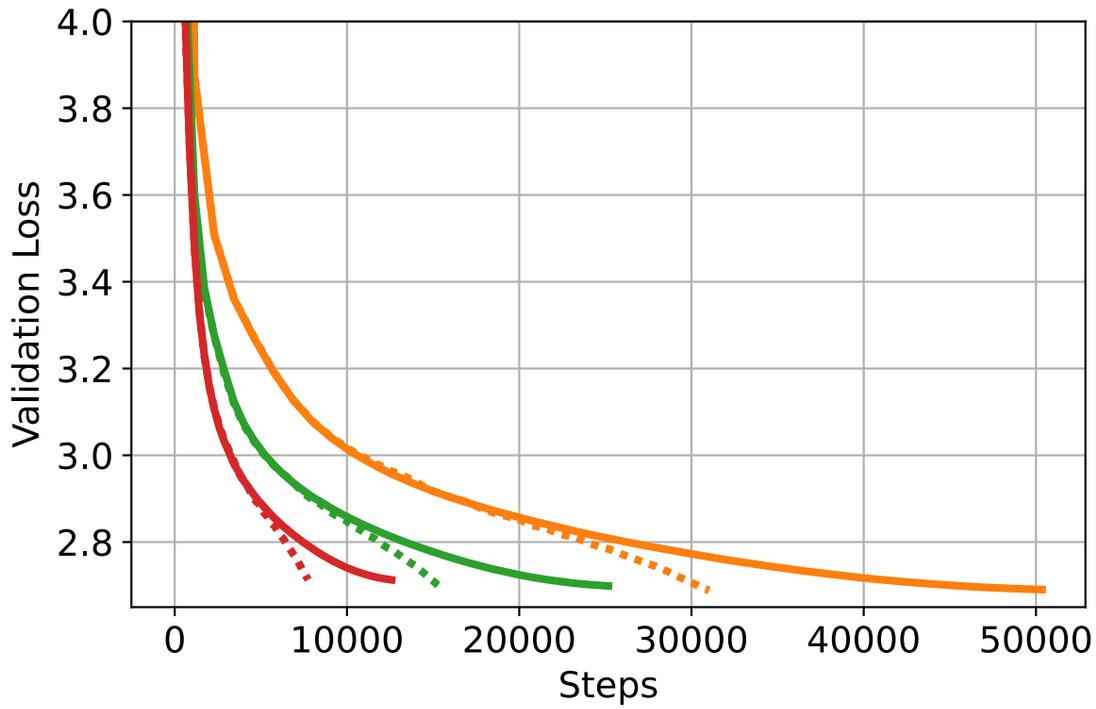


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*Seesaw*

- theory based, practice verified
- blackbox — no extra measures
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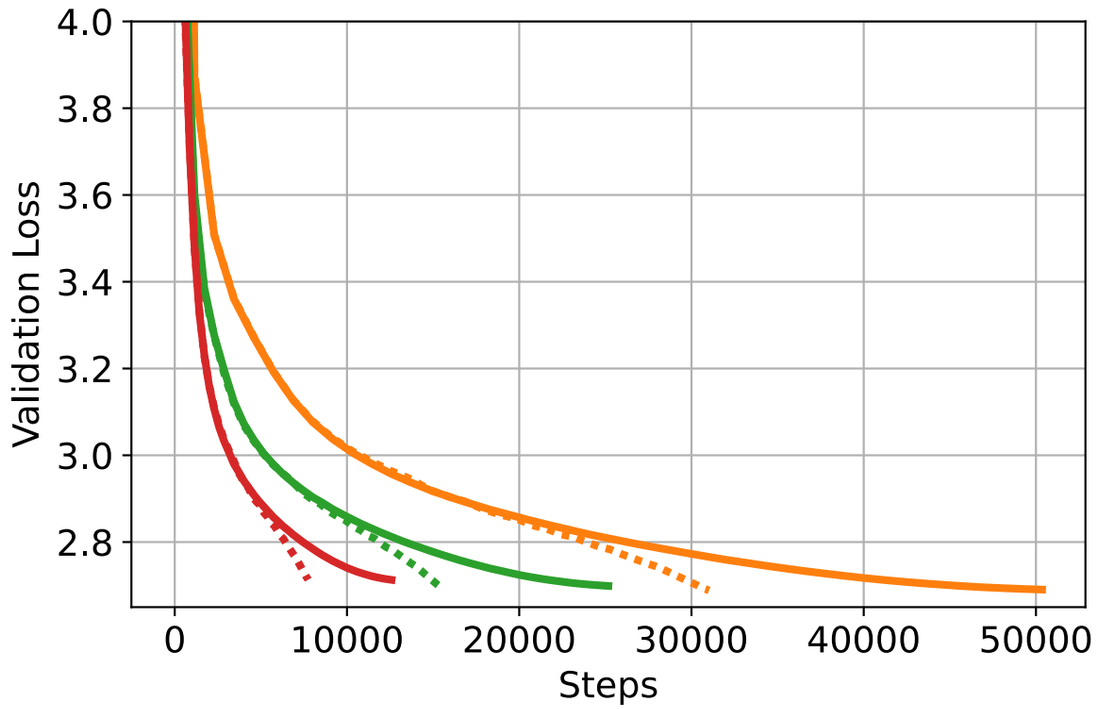


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simple, meaningful sandbox can be predictive!

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## Contribution 1: unstable optimization

large stepsize accelerates gradient descent in logistic regression

## Contribution 2: implicit regularization

gradient descent dominates ridge regression in linear regression

## Contribution 3: from theory to practice

principled parallelization method for training language models

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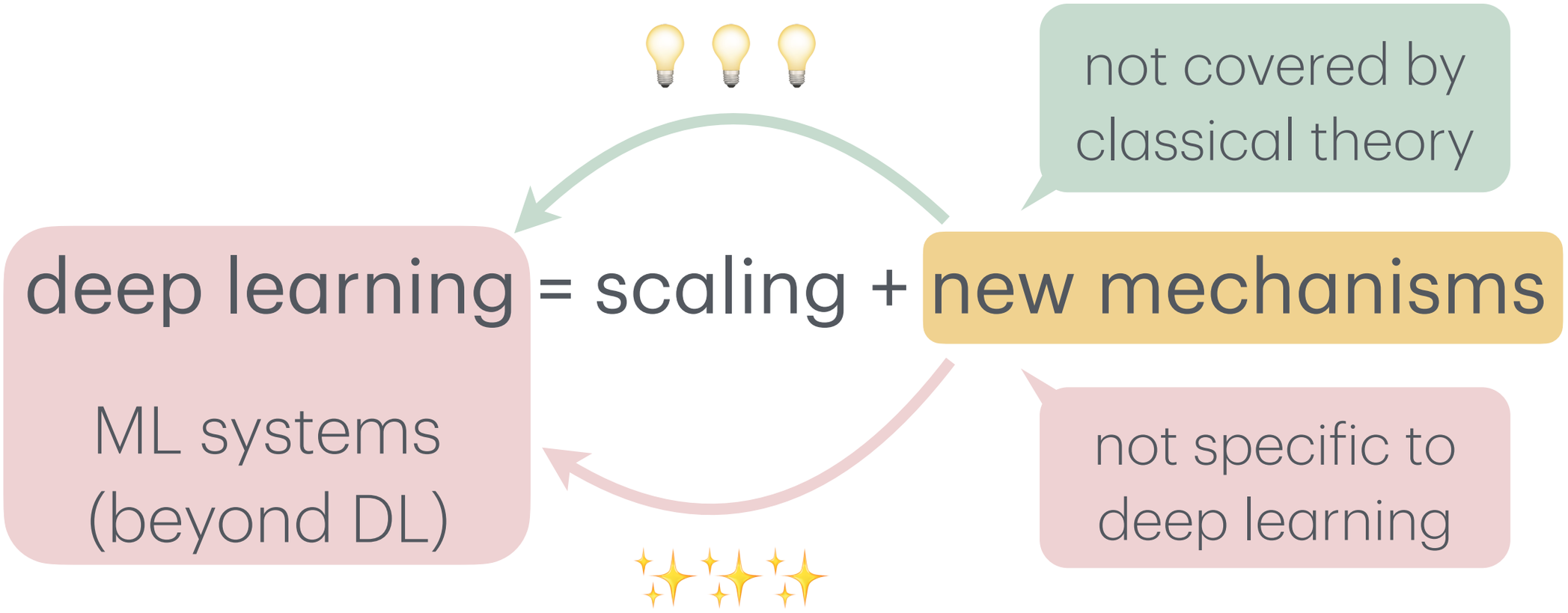


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# Summary

classical theory: conservative

“worst-case”, “stable”, …

optimization | statistics

# Summary

my research: less conservative

“instance-wise”, “unstable”, …

optimization x statistics

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model, data?
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for more discussions

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**stats → opt**

new criterion
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data reuse?

**opt → stats**

opt algorithm as estimator?

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| instability | stats → opt | practice |
|---|---|---|
| new technique?<br>model, data?<br>other instabilities?<br><br>*check out our tutorial for more discussions | new criterion<br>hyperparameter?<br>data reuse?<br><br>**opt → stats**<br>opt algorithm as estimator? | testbed<br>new question?<br>new sandbox?<br>other domain? |

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